PLANE ELASTIC STRUCTURES WITH UNILATERAL CONTACT

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Modeling of structural dynamics is an important task in the field of mechanical engineering. Elastic structures are usually modeled using the finite element method which is recommendable for complex geometries. In typical applications the finite elements used have linear properties. Friction and impact phenomena are strongly nonlinear effects that complicate the simulation considerably. Contact problems including deformable bodies are of particular importance, e.g. to analyze local deformations. There is a wide range of contact models that have been developed to date. The mathematically simplest approach is the use of penalty springs for the normal contact and tangential springs to approximate coulomb’s friction law. However, high spring stiffnesses are necessary to obtain feasible results which can lead to numerical problems. In the field of multibody dynamics where the contacting bodies are rigid and the number of contact points is relatively small the contact problem is often formulated as a linear complementarity problem (LCP). The LCP is a combinatorial problem which is usually solved using Lemke’s algorithm (see e.g. [1] or [2]). The mathematical description is more complex than for penalty springs, but there is nearly no penetration of the contacting bodies which is physically correct. Anyhow, there is only few work to find where the contact of deformable bodies and an LCP approach is combined, see e.g. [3] or [4]. The present work turns the attention on solving LCP’s set up for contact problems the way explained in [5]. The equation of motion is set up on velocity level and solved with a time stepping scheme. After integration over the time interval $\Delta t = [t^A, t^E]$ the equation of motion of the mechanical system with frictional contact reads as follows

$$M(\dot{q}^E - \dot{q}^A) - F^{(c)} \Delta t = W_N \tilde{F}_N + W_T \tilde{F}_T,$$  \hspace{1cm} (1)

where $M$ is the mass matrix, $F^{(c)}$ the active forces and $\tilde{F}_N$ and $\tilde{F}_T$ the normal and tangential impulses of the closed contacts projected onto the generalized coordinates $q$ via the transformation matrices $W_N$ and $W_T$, respectively. Neglecting the friction impulses and introducing the normal gap velocities $\dot{g}_N = W_N^T \dot{q}$ yields the LCP on velocity level for the frictionless contact which reads

$$\dot{g}_N^E = W_N^T M^{-1} W_N \tilde{F}_N + W_T^T M^{-1} F^{(c)} \Delta t - \dot{g}_N^A.$$  \hspace{1cm} (2)

Eq.(2) is a linear equation system of dimension $n$ which equals the number of closed contacts and exhibits $2n$ unknowns in $\dot{g}_N$ and $\tilde{F}_N$. The complementarity condition is

$$0 \leq \dot{g}_N^E \perp \tilde{F}_N \geq 0.$$  \hspace{1cm} (3)

Lemke’s algorithm, which is a pivoting algorithm especially developed to solve LCP’s, is commonly used to deal with complementarity problems. However, the mentioned LCP for the frictionless normal contact problem can be solved much easier. Since the LCP is set up only for contact points whose normal gap has been detected to be zero the normal gap velocities can be assumed to be zero. This assumption is correct for enduring contact and transition from separation to contact. The left hand side of eq.(2) is assumed to be zero. Thus, the LCP turns into a determined linear equation system for the $n$ unknown impulses. Anyhow, the side condition $\tilde{F}_N \geq 0$ has to be checked. A negative component of the solution vector indicates separation of the corresponding contact point in the next time step. Since the impulses must not be negative each negative component has to be set to zero manually. The above described way to solve an LCP for the normal contact problem can also be applied to frictional systems whose normal and tangential contact problem is decoupled. From a
mathematical point of view the contact problem is decoupled when $W^T W_T = 0$. The LCP for frictional contact as described in [4] can then be split up into the normal contact problem, cp. eq.(2), and the tangential contact problem which reads

$$
\begin{bmatrix}
\dot{g}^E_T \\
F^\oplus_T
\end{bmatrix} = A \begin{bmatrix}
\dot{g}^E_T \\
F^\ominus_T
\end{bmatrix} + \begin{bmatrix}
W^T M^{-1} F^{(e)} \Delta t + \dot{g}^A_T - \alpha \\
0
\end{bmatrix}
$$

(4)

with the abbreviations

$$A = \begin{bmatrix}
W^T M^{-1} W_T E \\
0
\end{bmatrix}
$$

(6)

$$\alpha = (W_N^T M^{-1} (W_N - W_T \mu))^{-1} W_N^T M^{-1} \dot{F}^{(e)} \Delta t + \dot{g}^A_N.
$$

(7)

Since coulombs friction law exhibits two corners two complementarity conditions are necessary to build up the LCP. The complementary quantities are defined by

$$F^\oplus_T = +F_T + \mu F_N$$

(8)

$$F^\ominus_T = -F_T + \mu F_N$$

(9)

$$\dot{g}^E_T = \dot{g}^E_T^\oplus - \dot{g}^E_T^\ominus.$$  

(10)

A detailed description how to derivate the LCP from the equation of motion in (1) can be found in [5]. Fig. 1 shows a plane elastic structure where the contact problem is decoupled and can be treated the way explained here. The system consists of four plane elements with altogether ten nodes. Each node exhibits two degrees of freedom describing the nodal displacements $u_x$ and $u_y$. 

A modal analysis of the finite element model yields the mass matrix and stiffness matrix of the system. The stiffness matrix multiplied with the generalized coordinates $q$ and a stiffness proportional damping matrix multiplied with the generalized velocities $\dot{q}$ build the active forces $F^{(e)}$. The system is dropped from a certain height and due to gravity the normal gaps vanish. Consequently, impact occurs when a gap function crosses zero. Eqs. (2) and (4) yield the contact impulses needed to compute the next time step. 

Fig. 2 shows exemplary the motion of node 1 defined in Fig. 1. The numerical results obtained with the LCP solving method explained above are verified with results obtained by using Lemke’s algorithm.

REFERENCES


