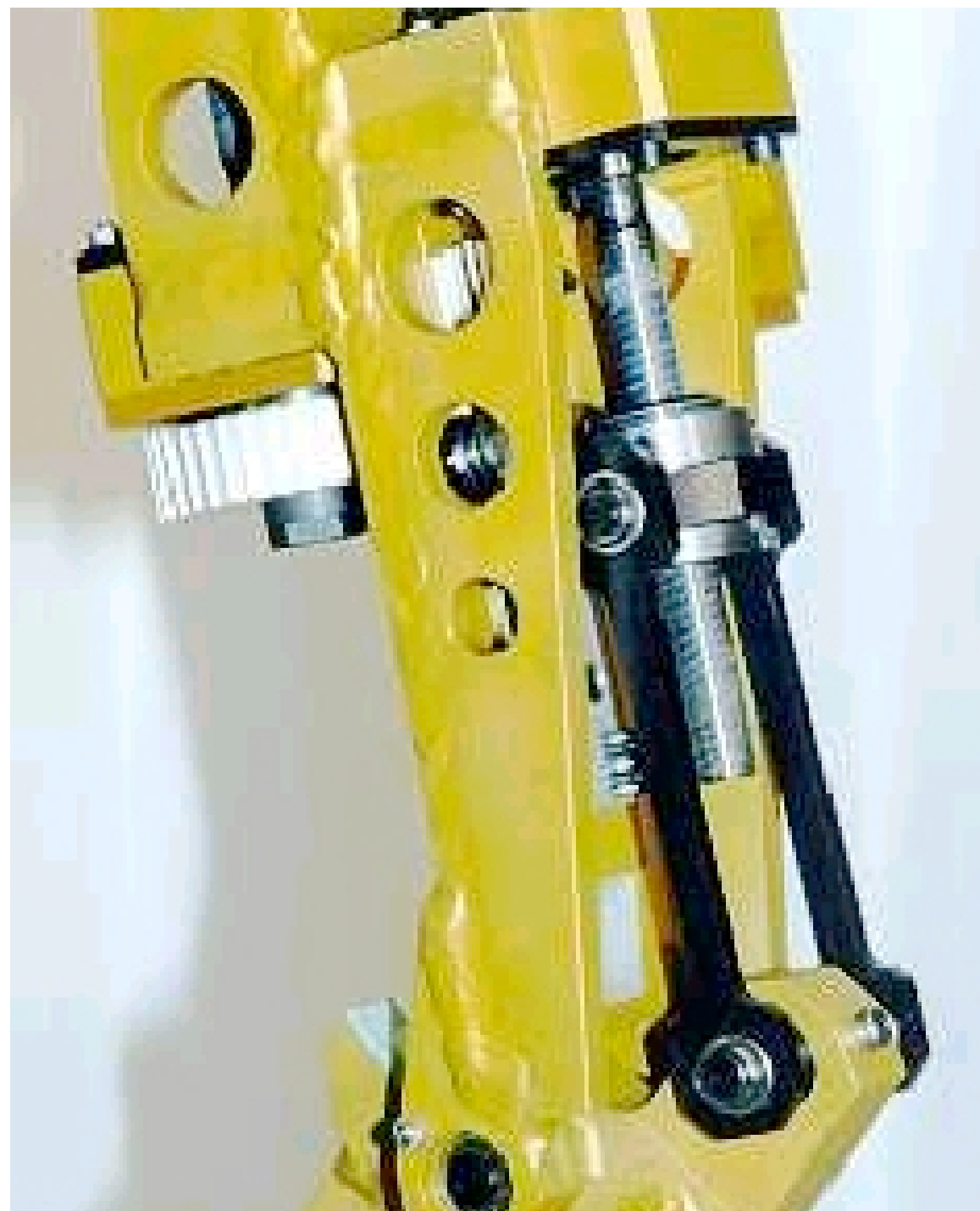


1 Introduction

- Goal: simulate dynamics of a mechanical system
 - with **unilateral contact**
 - and **Coulomb's friction** at contact points.
- Example: granular materials, robotics



Granular materials

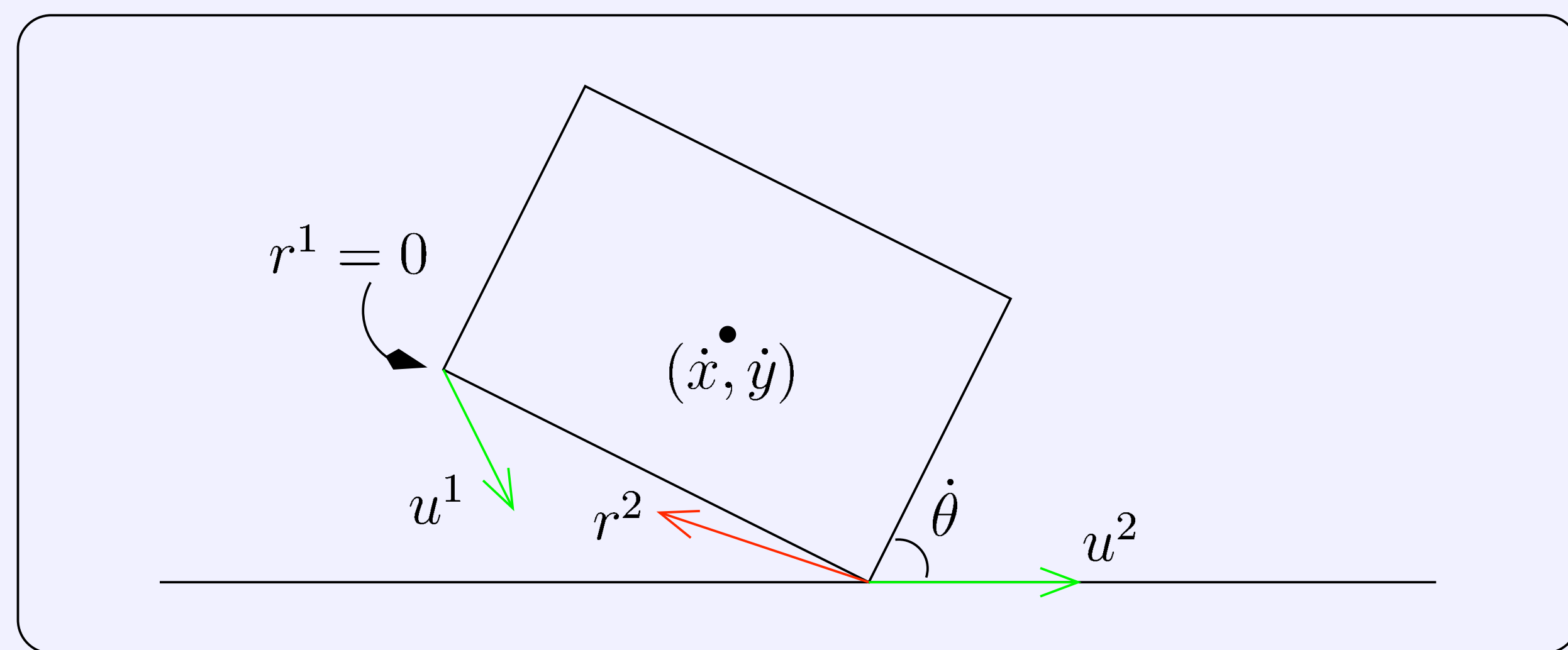


Robotics

- Mechanical system must have finitely many degrees of freedom
- No other nonlinearity than Coulomb's law

2 Unknowns

- Time is discretized
- We want to compute at each time step:
 - parameters (x, y, θ)
 - **generalized velocity** : $v = (\dot{x}, \dot{y}, \dot{\theta})$
 - **velocity** at contact points : $u = (u^1, u^2)$
 - **reaction** at contact points : $r = (r^1, r^2)$



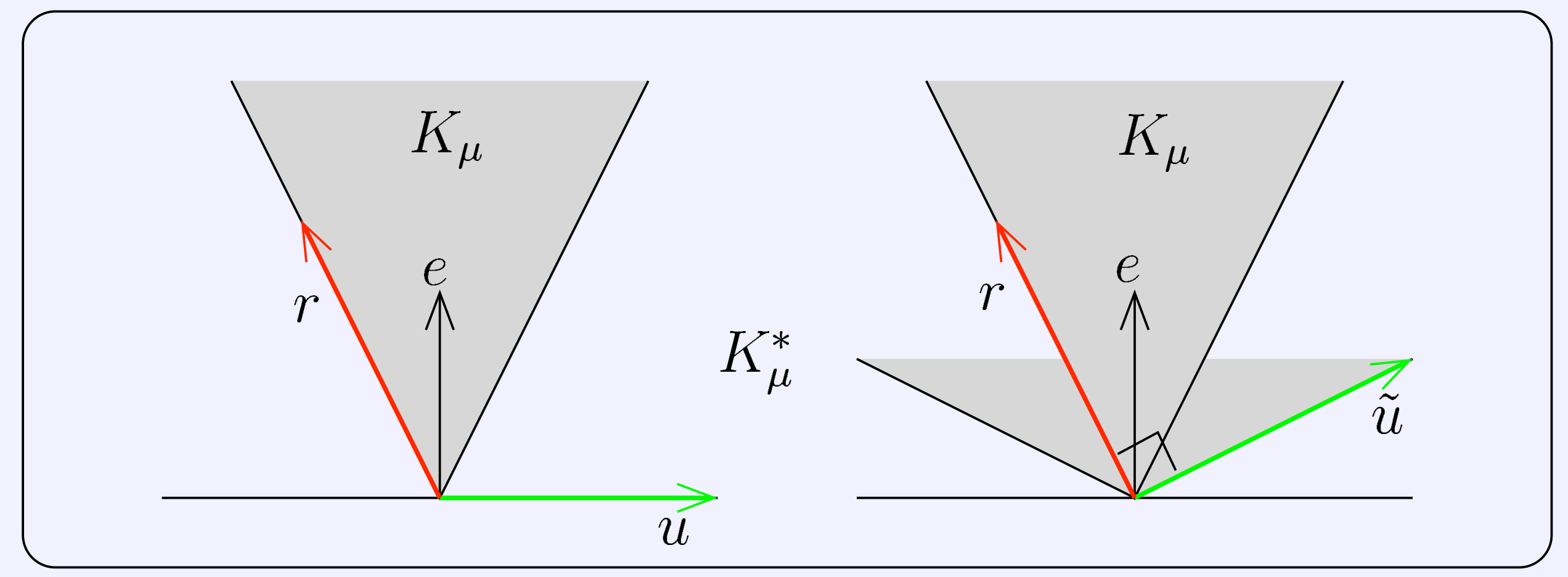
3 Coulomb's law

- Let $K_\mu = \{\|r_T\| \leq \mu r_N\} \subset \mathbb{R}^3$ (second order cone)
- Coulomb's law : usually formulated as 3-case **disjunction**:
 - **take off** : $r = 0$ and $u_N \geq 0$
 - **sticking** : $r \in \text{int}(K_\mu)$ and $u = 0$
 - **friction** : $r \in \partial K_\mu \setminus 0$, $u_N = 0$, u_T opposed to r_T .

4 Coulomb's law revisited

- Formulation as disjunction is not convenient
- More compact (and practical) formulation: **complementarity**

$$\begin{cases} \tilde{u} = u + \mu \|u_T\| e \\ K_\mu \ni r \perp \tilde{u} \in K_\mu^* \end{cases} \quad (1)$$



5 Formulation

- Altogether, we want to solve:

$$\begin{cases} Hr = Mv + f & \text{[Newton's law] (a)} \\ u = H^T v + Es & \text{[kinematics] (b)} \\ L \ni r \perp u \in L^* & \text{[Coulomb's law] (c)} \\ s^i = \|u_T^i\| \forall i & \text{(d)} \end{cases} \quad (2)$$

- $H, M \in \mathbb{S}_n^{++}$ (mass matrix), f, E are constant
- L is a product of several cones K_μ (one for each contact)

- Key observation : (a-c) are the **optimality conditions** of :

$$\begin{cases} \min J(v) := \frac{1}{2} v^T M v + f^T v & \text{(quadratic, strictly convex)} \\ H^T v + Es \in L^* & \text{(conic constraints)} \end{cases}$$

- The (equivalent) dual problem can also be used

6 Algorithm

- Consider s as a parameter
- Solve (2.a-2.c) as a **second order cone program** (SOCP)
- Adapt s iteratively in **damped Newton algorithm** to satisfy (2.d)
- Need to differentiate solution of SOCP with respect to s

7 Results

- Theoretical : simple proof of **existence of a solution**
- Numerical : **stability, very fast convergence**
 - Ex. : 3D, 200 degrees of freedom, 150 contacts, $\mu \leq 2$
 - only 3 iterations (ie 3 SOCP subproblems) are enough !

Iteration	1	2	3
Infeasibility in (2.d)	$1.1 * 10^3$	$4.8 * 10^{-3}$	$1.7 * 10^{-10}$

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