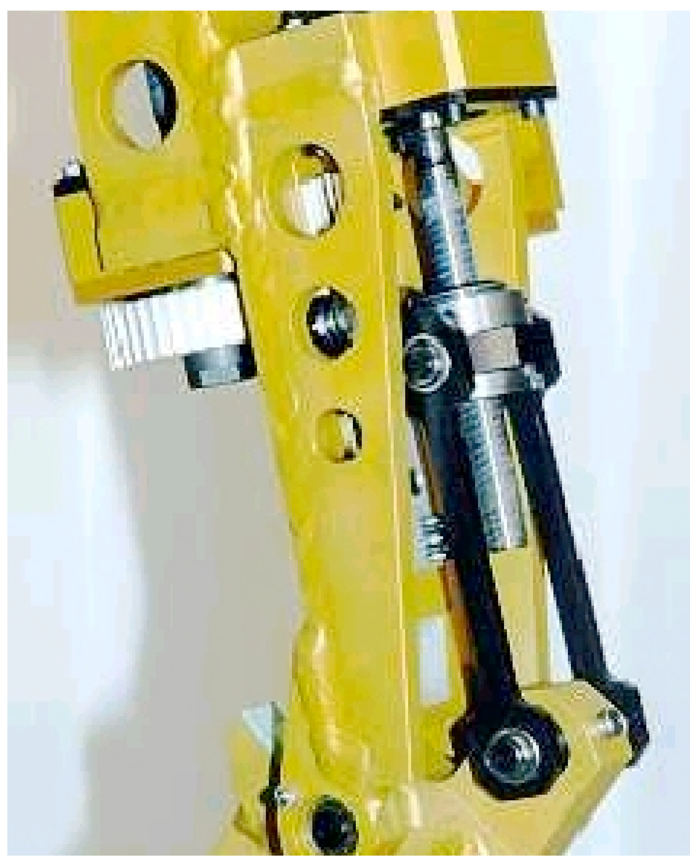
An algorithm for Coulomb's frictional contact

V. Acary, **F. Cadoux**, C. Lemaréchal, J. Malick

1 Introduction

- Goal: simulate dynamics of a mechanical system
  - with unilateral contact
  - and Coulomb's friction at contact points.
- Example: granular materials, robotics

Granular materials

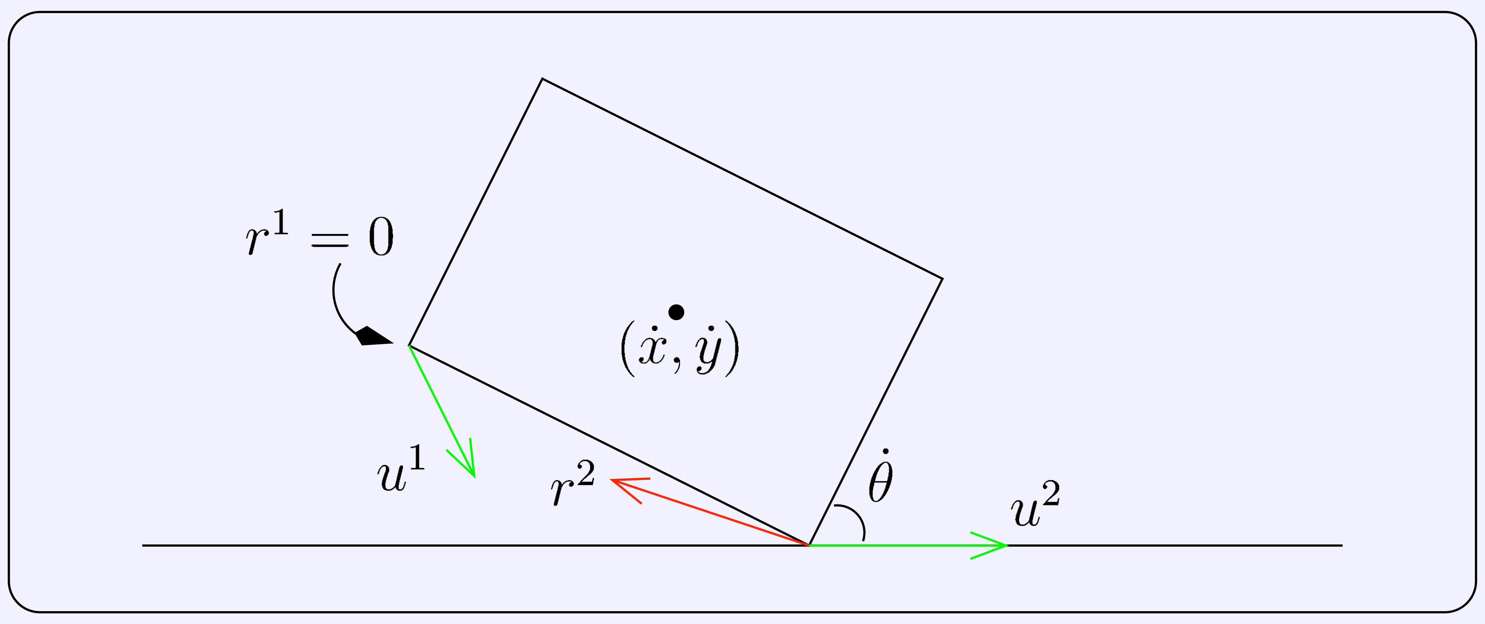


Robotics

- Mechanical system must have finitely many degrees of freedom
- No other nonlinearity than Coulomb's law

2 Unknowns

- Time is discretized
- We want to compute at each time step:
  - parameters (x, y, θ)
  - generalized velocity : $v = (\dot{x}, \dot{y}, \dot{\theta})$
  - velocity at contact points : $u = (u^1, u^2)$
  - reaction at contact points : $r = (r^1, r^2)$



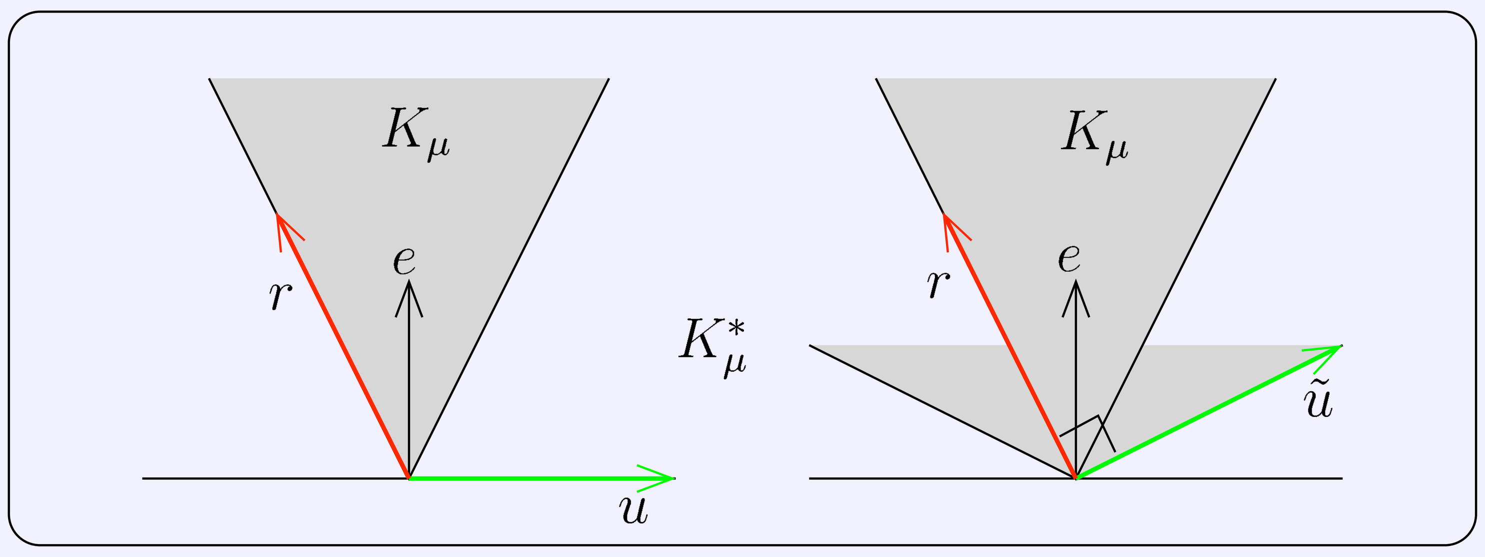
3 Coulomb's law

- Let $K_\mu = \{\|r_T\| \leq \mu r_N\} \subset \mathbf{R}^3$ (second order cone)
- Coulomb's law : usually formulated as 3-case disjunction:
  - take off : $r = 0$ and $u_N \geq 0$
  - sticking : $r \in int(K_\mu)$ and $u = 0$
  - friction : $r \in \partial K_\mu \setminus 0,\ u_N = 0,\ u_T$ opposed to r_T.

4 Coulomb's law revisited

- Formulation as disjunction is not convenient
- More compact (and practical) formulation: complementarity

$$\begin{cases} \tilde{u} = u + \mu \|u_T\| e \\ K_\mu \ni r \perp \tilde{u} \in K_\mu^* \end{cases} \qquad (1)$$



5 Formulation

- Altogether, we want to solve:

$$\begin{cases} Hr = Mv + f & \text{[Newton's law]}\ (a) \\ u = H^\top v + Es & \text{[kinematics]}\ (b) \\ L \ni r \perp u \in L^* & \text{[Coulomb's law]}\ (c) \\ s^i = \|u_T{}^i\|\ \forall i & (d) \end{cases} \qquad (2)$$

  - $H, M \in \mathbf{S}_n^{++}$ (mass matrix), f, E are constant
  - L is a product of several cones K_μ (one for each contact)
- Key observation : (a-c) are the optimality conditions of :

$$\begin{cases} \min J(v) := \frac{1}{2} v^\top M v + f^\top v & \text{(quadratic, strictly convex)} \\ H^\top v + Es \in L^* & \text{(conic constraints)} \end{cases}$$

- The (equivalent) dual problem can also be used

6 Algorithm

- Consider s as a parameter
- Solve (2.a-2.c) as a second order cone program (SOCP)
- Adapt s iteratively in damped Newton algorithm to satisfy (2.d)
- Need to differentiate solution of SOCP with respect to s

7 Results

- Theoretical : simple proof of existence of a solution
- Numerical : stability, very fast convergence
  - Ex. : 3D, 200 degrees of freedom, 150 contacts, $\mu \leq 2$
  - only 3 iterations (ie 3 SOCP subproblems) are enough !

Iteration	1	2	3
Infeasibility in (2.d)	$1.1 * 10^3$	$4.8 * 10^{-3}$	$1.7 * 10^{-10}$

Contact: *florent.cadoux@inrialpes.fr*

[1] De Saxcé G., Feng Z.Q. *The bipotential method: a constructive approach to design the complete contact law with friction* In Math. Comput. Mod., 1998
[2] Moreau J.-J. *Modélisation et simulation de matériaux granulaires* In Actes du CANUM 2003
[3] Alart P., Curnier P. *A mixed formulation for frictional contact prone to Newton-like solution methods* In Comp. Meth. in App. Mech. and Eng., 1991
[4] Christensen P.W., Klarbring A., Pang, J.S., Strömberg N. *Formulation and comparison of algorithm for frictional contact problems* In Int. J. Numer. Meth. Engng., 1998