PATH-TRACKING FOR TRACTOR-TRAILERS WITH HITCHING OF BOTH THE ON-AXLE AND THE OFF-AXLE KIND

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Abstract . **Results relevant to path-tracking control for a tractor-2-trailers vehicle with one coupling joint 'on axle' and the other 'off axle', are developed. In the case of off axle hitchings with a negative offset, these results are based on input/output linearization and extend controllers' design procedures already available for vehicles of which the coupling joints are all 'on axle', or all 'off axle'. In the case of off axle hitchings with a positive offset, exact linearization is no longer applicable and the problem is solved using Jacobian linearization. Convergence and robustness properties of the ensuing controllers are illustrated by means of simulation. Extension of these results to more general tractor-n-trailers vehicles is discussed.**

Index terms-- **Tractor-trailer, virtual tractor, path-tracking, on-axle, off-axle, hitching, positive offset, negative offset, maneuvers**

I. INTRODUCTION

In discussing automatic execution of low speed maneuvers on the part of tractor-trailers-like vehicles (e.g.: [1]-[10]) it is important to consider the specific nature of the hitchings that connect the various units of the articulated vehicle. A hitching is called *on axle* when the joint connecting two units is located at the center-point of the rear axle of the unit that is closer to the tractor. A hitching is called *off axle with a negative offset* (or simply off axle) when the coupling joint is located beyond the center point of the rear axle; it is called *off axle with a positive offset* when the joint is located in front the center point of the rear axle.

For tractor-trailers with on-axle hitching path-tracking and stabilization controllers have been available for now some time ([7], [9], [10]). Common features of these controllers is choice of a guide-point located at the center-point of the rear axle of the last trailer and adoption of exact input-state linearization techniques.

For tractor-trailers with "off-axle" hitching (with a negative offset) similar results have been obtained more recently ([4], [5]). In this latter case, exact input-state linearization has been replaced by input-output linearization and choice of guide-point has been made dependent on the particular maneuver (forward, reverse) to be implemented. For vehicles with hybrid hitching (some of the couplings are on-axle and some other off-axle) little has been done up to now and no results are presently available in their regard.

The objective of the present paper is to study path-tracking controllers for this latter kind of vehicles. As a first prototype of a hybrid vehicle, we consider a tractor with two trailers of which the first trailer has an on-axle hitching while the second has an off-axle hitching with a negative offset (figure 1). As a second prototype vehicle, we consider a tractor connected to trailer with an off-axle hitching with a positive offset (figure 2). In both cases the control task is to modulate linear and angular velocities of the tractor so that a selected 'guide-point' of the vehicle follows an assigned path both in forward and reverse direction.

II. NOMENCLATURE

The specific vehicle under consideration is represented in figure 1. The symbols have the following meaning:

x_i, y_i, θ_i : position and orientation of rear axle center-point of vehicle's ith unit, i=1, 2, 3;

ϕ_j : orientation of jth unit relative to (j-1)th, j= 2, 3;

v_1, Ω_1 : longitudinal and angular velocities of the tractor;

v_i, Ω_i : longitudinal and angular velocities of ith trailer, i= 2, 3,… n;

ℓ_j : wheelbase of ith trailer, j= 2, 3,… n;

c_i : off-axle offset of ith unit, i=1, 2,… n-1;

ℓ_{osi} : path-tracking error lateral offset (distance between guide-point and assigned path), i=1, n.

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III. TRACTOR2TRAILERS VEHICLE WITH ONE ‘ON AXLE’ AND ONE ‘OFF AXLE’ HITCHING

The design of a path tracking controller entails determination of longitudinal and angular velocities of the tractor, (v_1, Ω_1), that make a selected ‘guide-point’ of the vehicle follow an assigned path with assigned velocity both in the forward and backward direction. In what follows we will discuss this problem by confining attention to slow speed maneuvers, a slippage-free motion, and a cinematic description of the vehicle. In agreement with Bolzern & alias 2002, selection of the guide-point is dependent on the direction of motion: in the case of forward motion, the guide-point is located on the center of the rear axle of the tractor; in reverse motion, it is located on the center of the rear axle of the last trailer (see figure 1). For simplicity of exposition, we will consider straight-line paths to be tracked with an assigned constant velocity, v_{1d} .

III.A The case of forward motion

Indicating with ℓ_{os1} the path-tracking lateral offset (distance between guide-point and path to be tracked), the problem is to determine longitudinal and angular velocities (v_1^*, Ω_1^*) such that, by imposing $v_1 = v_1^*, \Omega_1 = \Omega_1^*$, one has

$$\begin{array}{l} \lim\ v_1(t) = v_{1d};\quad \lim\ l_{os1}(t)=0; \\ \lim\ \theta_1(t)=0\ \ \lim\ \varphi_1(t)=0;\ \ \lim\ \varphi_2(t)=0. \end{array} \tag{0.1}$$

Proposition 1.

i. In forward motion, a locally stable path-tracking controller is given by

$$\begin{aligned} &v_1^* = v_{1d} \\ &\Omega_1^* = -\frac{1}{v_{1d}\cos\theta_1}(k_{pf}\ell_{os1} + k_{vf}\dot{\ell}_{os1}) \end{aligned} \tag{0.2}$$

ii. The lateral offset dynamics associated with this controller is described by

$$\ddot{\ell}_{os1} = -k_{pf}\ell_{os1} - k_{vf}\dot{\ell}_{os1}; \tag{0.3}$$

iii. the zero dynamics of the feedback system is locally stable and described by

$$\begin{aligned} &\dot{\phi}_1 = -\frac{1}{\ell_2}v_{1d}\sin\phi_1 \\ &\dot{\phi}_2 = -\frac{1}{\ell_3}v_{1d}\cos\phi_1\sin\phi_2 - \frac{1}{\ell_2}v_{1d}\sin\phi_1 \end{aligned} \tag{0.4}$$

III.B The case of reverse motion

In reverse motion we measure the lateral path-tracking offset in terms of distance between guide-point now located on the second trailer and the assigned path (ℓ_{os3}). In this case, the control design entails determining tractor’s longitudinal and angular velocities (v_1^*, Ω_1^*) such that by imposing $v_1 = v_1^*, \Omega_1 = \Omega_1^*$ one has

$$\begin{array}{l} \lim\ v_3(t) = v_{3d};\quad \lim\ l_{os3}(t)=0; \\ \lim\ \theta_1(t)=0;\ \lim\ \varphi_1(t)=0;\ \ \lim\ \varphi_2(t)=0. \end{array} \tag{0.5}$$

Proposition 2.

i. In a reverse motion, a locally stable path-tracking controller is given by

$$\begin{aligned} &v_1^* = \left(v_{3d}\cos\phi_2 - \ell_3\sin\phi_2\Omega_3\right)\cos\phi_1 - \ell_2\sin\phi_1\Omega_2 \\ &\Omega_1^* = \frac{\ell_2\cos^2\phi_1\dot{\Omega}_2 + \dot{v}_2\cos\phi_1\sin\phi_1}{v_2(\cos^2\phi_1 - \sin^2\phi_1) - 2\ell_2\Omega_2\cos\phi_1\sin\phi_1} \end{aligned} \tag{0.6}$$

where

$$\begin{aligned} &\dot{\phi}_2 = \frac{1}{c_2}\left\{v_{3d}\sin\phi_2 + \Omega_3(c_2 + \ell_3\cos\phi_2)\right\} \\ &\dot{v}_2 = -v_{3d}\sin\phi_2\dot{\phi}_2 \\ &\dot{\Omega}_2 = \frac{1}{c_2\cos\phi_2}[-\ell_3\dot{\Omega}_3^*\cos^2\phi_2 + 2\Omega_3\ell_3\dot{\phi}_2\sin\phi_2\cos\phi_2 \\ &-v_3\dot{\phi}_2(\cos^2\phi_2 - \sin^2\phi_2) + \Omega_2 c_2\dot{\phi}_2\sin\phi_2 \\ &\dot{\Omega}_3^* = v_{3d}\sin\theta_3\Omega_3^2 - \frac{1}{v_{3d}\cos\theta_3}(k_{pr}\ell_{os3} + k_{vr}\dot{\ell}_{os3} + k_{ar}\ddot{\ell}_{os3}) \end{aligned}$$

.(0.7)

ii. The path-tracking lateral offset dynamics associated with this controller is described by

$$\dddot{\ell}_{os3} = -k_{pr}\ell_{os3} - k_{vr}\dot{\ell}_{os3} - k_{vr}\ddot{\ell}_{os3}; \tag{0.8}$$

iii. the zero dynamics of the feedback system is locally stable and described by

$$\begin{aligned} &\dot{\phi}_2 = \frac{1}{c_2}\left\{v_{3d}\sin\phi_2\right\} \\ &\dot{\phi}_1 = \frac{v_{3d}\sin^2\phi_2\cos\phi_1\sin\phi_1}{c_2(\cos^2\phi_1 - \sin^2\phi_1)\cos\phi_2} \end{aligned} \tag{0.9}$$

III.C Simulation Results.

The control strategy embodied in propositions 1 and 2 has been applied to a simulated model of a tractor-2-trailer vehicle with the following geometrical parameters:

$$\ell_2 = .58m; \ell_3 = .63m; c_1 = 0; c_2 = .5m.$$

In what follows we will report on some of the results that we have obtained in correspondence to tracking in reverse motion with an assigned speed v_{1d}=5m/sec. The poles and subsequent gains characterizing the dynamics of the lateral offset dynamics the guide-point (located on the rear axle of the second trailer) are as follows

$$\begin{aligned}
&\dddot{\ell}_{os3} + K_3\ddot{\ell}_{os3} + K_2\dot{\ell}_{os3} + K_1\ell_{os3} = 0\\
&p_1 = -0.7 + 0.7 * i \quad p_2 = -0.7 - 0.7 * i\\
&p_3 = -1\\
&K_3 = -(p_1 + p_2 + p_3)\\
&K_2 = p_1p_2 + p_1p_3 + p_2p_3\\
&K_1 = -p_1p_2p_3\\
&\text{d'où}\\
&K_1 = 1\\
&K_2 = 2.41\\
&K_3 = 2.41
\end{aligned} \tag{1.1}$$

A great number of simulation results have shown that dynamic behavior of the path-tracking offsets corresponds precisely to the behavior predicted by eqn (0.8). It was found that the region of convergence of these same offsets which is adequate for most of the practical applications that can be envisioned for this kind of controllers (parking maneuvers in restricted space). It was also found that the system is quite robust to parameter uncertainty.

IV. The case of off-axle hitching with a positive offset

The above development, in concordance with most (all?) other published developments about path-tracking and stabilization of tractor-trailers, is not applicable to a vehicle with a coupling joint located between rear and front axles ("positive off-axle hitching"). The problem is that the technique of exact linearization, of both the input-state and input-output kind, is not applicable to vehicles with positive off-axle hitching (Bolzern et al. 2001). The problem can in this case be solved by using Jacobian linearization. In what follows we illustrate the procedure by considering a tractor-trailer with a geometric configuration as in figure 2.

IV.A Vehicle Model.

Adopting usual no slippage hypotheses, the vehicle can be modeled in terms of the following equations

$$\dot{v}_{u1} = U_1$$
$$\dot{\Omega} = U_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\theta}_1 \\ \dot{\Phi} \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) & 0 \\ \sin(\theta_1) & 0 \\ 0 & 1 \\ -\dfrac{\sin(\theta_1)}{l_2} & \dfrac{c_1.\cos(\theta_1)}{l_2} - 1 \end{bmatrix} . [\, v_{u1} \quad \Omega \,] \tag{1.2}$$

$$\begin{aligned}
\mathbf{x_2} &= \mathbf{x_1 + c_1\cos(\theta_1) - l_2\cos(\theta_1 + \Phi)}\\
\mathbf{y_2} &= \mathbf{y_1 + c_1\sin(\theta_1) - l_2\sin(\theta_1 + \Phi)}\\
\boldsymbol{\theta_2} &= \boldsymbol{\theta_1 + \Phi}
\end{aligned}$$

where U_1 and U_2 represent propulsion and steering controls, (x_1, y_1) the position of the center-point of the tractor's rear axle, (x_2, y_2) the position of the center-point of the trailer's rear axle.

IV.B Design of the Path-Tracking Controller.

The problem is to determine controls U_1 and U_2 that make the center-point of the tractor's rear axle (guide-point) follow a given path with an assigned velocity. For simplicity of exposition we consider this problem in the context of a path made of a circle of radius R, to be followed with a constant velocity, v_{ud}. By measuring path-tracking error by means of lateral, orientation and velocity path-tracking offsets defined by

$$\begin{aligned}
&\boldsymbol{\ell_{os}} := \mathbf{sqrt[(x_1 - x_{1d})^2 + (y_1 - y_{1d})^2]}\\
&\mathbf{v_{os}} := \mathbf{v_u - v_{ud}}\\
&\boldsymbol{\Phi_{os}} := \boldsymbol{\Phi - \Phi}\mathbf{d}\\
&\boldsymbol{\theta_{os}} := \boldsymbol{\theta - \theta_d}
\end{aligned} \tag{1.3}$$

the problem becomes equivalent to determining U_1 and U_2 so that

$$\mathbf{lim}_{t\to\infty}\ [\boldsymbol{\ell_{os}}\ \mathbf{v_{os}}\ \boldsymbol{\Phi_{os}}\ \boldsymbol{\theta_{os}}] = \mathbf{0.}$$

To find such a control law we start by noting that the offset dynamics can be described in terms of the following equations

$$\dot{\ell}_{os} = \mathbf{v_u} \sin(\theta_{os})$$
$$\dot{\mathbf{v}}_{os} = \dot{\mathbf{v}}_u - \dot{\mathbf{v}}_{ud} = \mathbf{U1} \tag{1.4}$$
$$\ddot{\theta}_{os} = \ddot{\theta} - \ddot{\theta}_d = \mathbf{U2}$$
$$\dot{\Phi}_{os} = \frac{-1}{l_2}[(v_{os} + v_{ud}).\sin(\Phi_{os} + \Phi_d) +$$
$$(\dot{\theta}_{os} + \dot{\theta}_d)[l_2 - c_1.\cos(\Phi_{os} + \Phi_d)]]$$

where

$$\alpha = \frac{\text{-Vud}}{\mathbf{l_2}}\cos(\Phi_d) - \frac{\mathbf{c_1}}{\mathbf{l_2}}\dot{\theta}_d \sin(\Phi_d)$$
$$\beta = -\frac{\mathbf{l_2} - \mathbf{c_1}\cos(\Phi_d)}{\mathbf{l_2}} \tag{1.5}$$

By introducing the notation

$$\mathbf{X_{os}} := [\mathbf{v_{os}} \quad \theta_{os} \quad \Phi_{os} \quad \ell_{os} \quad \dot{\theta}_{os}]',$$

and by applying local linearization it follows

$$\dot{X}_{os} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha & 0 & \beta \\ 0 & v_{ud} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} X_{os} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}. \tag{1.6}$$

From these equations, one can see that $\lim_{t\to\infty} [\mathbf{v_{os}}] = 0$ can be ensured by selecting $\mathbf{U_1} = -\mathbf{k_{p1}}\mathbf{v_{os}} - \mathbf{k_{p2}}\int \mathbf{v_{os}}\mathbf{dt}$. This longitudinal control law provides a dynamics described by $\ddot{\mathbf{v}}_{os} + \mathbf{k_{p1}}\dot{\mathbf{v}}_{os} + \mathbf{k_{p2}}\mathbf{v_{os}} = 0$ and therefore by poles $\mathbf{p_{11}},\ \mathbf{p_{12}}$ given by

$$\mathbf{k_{p1}} = -(\mathbf{p_{11}} + \mathbf{p_{12}}) \text{ and } \mathbf{k_{p2}} = \mathbf{p_{11}}\mathbf{p_{12}}.$$

To ensure $\lim_{t\to\infty} [\ell_{os}\ \Phi_{os}\ \theta_{os}] = 0$ we select

$$\mathbf{U_2} = \mathbf{k_{s1}}\theta_{os} + \mathbf{k_{s2}}\Phi_{os} + \mathbf{k_{s3}}\ell_{os} + \mathbf{k_{s4}}\dot{\theta}_{os} + \mathbf{k_{s5}}\int \ell_{os}\mathbf{dt}$$

This lateral control law provides a dynamics described (in the Laplace domain) by

$$\mathbf{s}^2\theta_{os} = (\mathbf{k_{s1}} + \mathbf{k_{s3}}(\frac{\mathbf{v_{ud}}}{\mathbf{s}} + 1) + \mathbf{k_{s4}}\mathbf{s})\theta_{os} + \mathbf{k_{s2}}\Phi_{os}$$

and therefore by poles $\mathbf{p_{21}},\ \mathbf{p_{22}}\ \mathbf{p_{23}},\ \mathbf{p_{24}},\mathbf{p_{25}},$ defined by

$$-\alpha - k_{s4} = -p_{25} - p_{23} - p_{22} - p_{21} - p_{24};$$
$$-k_{s1} - k_{s3} - k_{s5} - \beta k_{s2} + k_{s4}\alpha = p_{22}p_{24} + p_{21}p_{25} + p_{21}p_{24} + p_{22}p_{25} + p_{23}p_{24} + p_{21}p_{22} + p_{22}p_{23} + p_{23}p_{25} + p_{21}p_{23} + p_{24}p_{25};$$
$$k_{s1}\alpha + k_{s3}(\alpha - v_{ud}) + k_{s5}(\alpha - 1) = -p_{23}.p_{24}.p_{25} - p_{22}p_{24}p_{25} - p_{22}p_{23}p_{25} - p_{21}p_{24}p_{25} - p_{21}p_{23}p_{25} - p_{21}p_{22}p_{25} - p_{21}p_{23}p_{24} - p_{21}p_{22}p_{24} - p_{21}p_{22}p_{23} - p_{22}p_{23}p_{24};$$
$$k_{s5}\alpha - k_{s5}Vud + k_{s3}v_{ud}\alpha = p_{22}p_{23}p_{24}.p_{25} + p_{21}p_{22}p_{24}p_{25} + p_{21}p_{22}p_{23}p_{25} + p_{21}p_{22}p_{23}p_{24} + p_{21}p_{23}p_{24}p_{25};$$
$$k_{s5}v_{ud}\alpha = -p_{21}p_{22}p_{23}p_{24}p_{25}. \tag{1.7}$$

IV.3 Simulation Results

To investigate the behavior of the proposed controller, consider the problem of having a tractor-trailer track with a constant speed of 5m/s a circular path with radius equal to 5 m. Let the vehicle have a geometry determined by a positive off-axle hitch offset corresponding to $\mathbf{c_1} = \mathbf{1m}$, and by a trailer's wheel-base corresponding to $\mathbf{l_2} = \mathbf{2\ m}$.

Three versions of the proposed controller have been analyzed: a first version employs static state-feedback without integrating action on the lateral offset; a second version adds an integrator action; a third version is equal to the first for large lateral offsets, to the second for small offsets.

The poles for the first version are chosen as given by $\mathbf{p_{21}} = \mathbf{p_{22}} = \mathbf{p_{23}} = \mathbf{p_{24}} =$-2.5(1$\pm$j). This leads to gains K_{av} = [12.4339 -12.0664 12.0655 7.4120] for forward maneuver, and K_{ar} = [-85.0699 -304.1088 12.0655 12.3870] for backward maneuver. The poles for the second version are chosen as given by $\mathbf{p_{21}} = \mathbf{p_{22}} = \mathbf{p_{23}} = \mathbf{p_{24}} =$-2 (1$\pm$j) and $\mathbf{p_{25}}$=-2. The ensuing gains are K_{av} =[-8.1738 0.6468 -3.3644 -5.1569 -2.5600] for forward maneuver, and K_{ar} = [42.6961 172.8112 -6.4364 -10.1569 -2.5600] for backward maneuver.

As illustrated in figures 3, simulation results suggest the behavior of the various versions of the controller to work satisfactorily. In particular, the static state feedback version shows a faster response time and a larger region of stability. In this version, however, one finds a lateral path-tracking error proportional to the curvature radius of the path plus the need to use curvature-path dependent gains. This error and this dependence can be eliminated by equipping the controller with an integral action proportional to the lateral error. This improvement, however, comes at the cost of a reduction of the stability region. Adoption of a hybrid controller whereby integral action is only applied for small errors appears to provide elimination of steady state error without the cost of reduction in the stability region and the need of path-dependent feedback gains. Sensitivity of the controllers to trailer's wheelbase was also investigated. It was found that the controller can operate satisfactorily with an uncertainty of the order of 10 % of the wheelbase value.

CLOSURE

Available exact linearization results relevant to path-tracking control for tractor-trailers-like vehicles are usually confined to vehicles of which the hitchings are all on axle or all off axle with a negative offset. Propositions 1 and 2 extend these results to vehicles of which some of the hitchings are on axle and some are off axle (with a negative offset). Although, for simplicity of exposition, these propositions are stated and proved in the specific case of a tractor-2-trailers vehicle and of straight-line paths to be followed with a constant velocity, the approach that we have adopted is of a general nature and readily applicable to more complex tractor-n-trailers vehicles and more diversified paths. For example, for a vehicle made of a tractor towing n-1 trailers of which m coupling joints are on axle and n-1-m are off axle, the essence of this generalization can be formalized as follows.

Proposition 3.

i. In forward motion, with guide-point chosen on the center-point of the tractor's rear axle, a locally stable path tracking controller is described by equations (0.2).

ii. In reverse motion, with guide-point chosen on the center-point of the last trailer's rear axle, a locally stable path-tracking controller can be determined as follows. The longitudinal velocity of the tractor , v_1 * , is chosen such that $v_1 = v_1{}^*$ implies $v_n = v_{nd} := v_{1d}$. The angular velocity $\Omega_1{}^*$ is chosen such that $\Omega_1 = \Omega_1{}^*$ implies $\left[\frac{d\Omega_n{}^m}{dt^m}\right] = \left[\frac{d\Omega_n{}^m}{dt^m}\right]^*$ where $\left[\frac{d\Omega_n{}^m}{dt^m}\right]^*$ is such that

$$\frac{d\ell_{os3}{}^{2+m}}{dt^{2+m}} = -k_1\ell_{os3} - k_2\dot{\ell}_{os3} - k_3\ddot{\ell}_{os3} - \dots\dots\dots - k_{2+m}\frac{d\ell_{os3}{}^{1+m}}{dt^{1+m}}$$

. (2.1)

In the case of articulated vehicles of which one or more off axle hitchings are characterized by a positive offset, exact linearization is no longer applicable and the path tracking problem must be approached using Jacobian linearization. A typical procedure to carry out such an approach and typical ensuing results have been illustrated by considering a tractor-trailer vehicle with the required pathological geometry.

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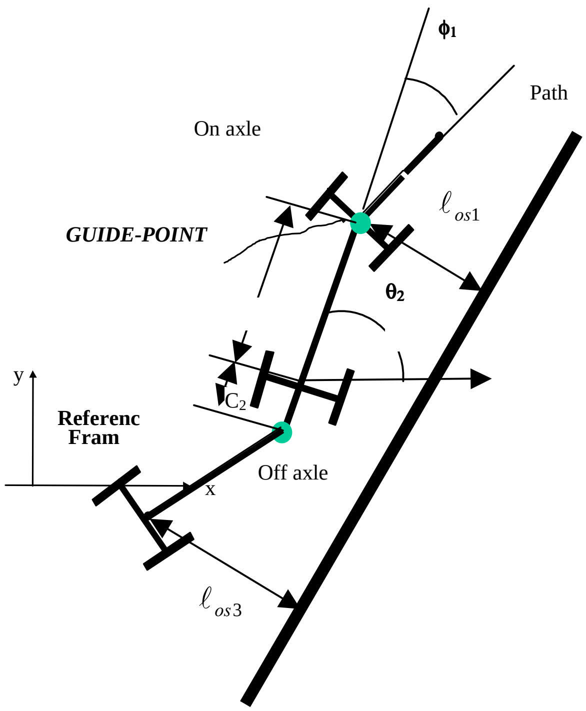


Figure 1: Tractor-2-trailers under study

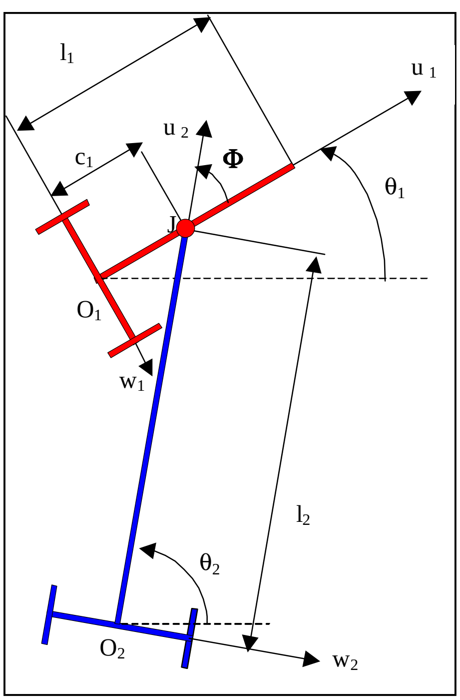
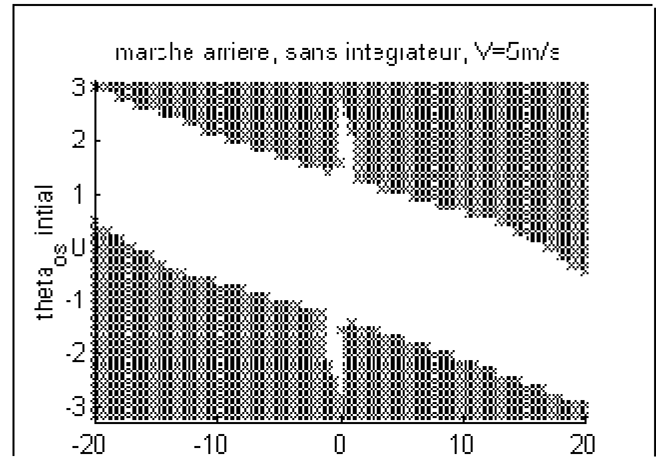
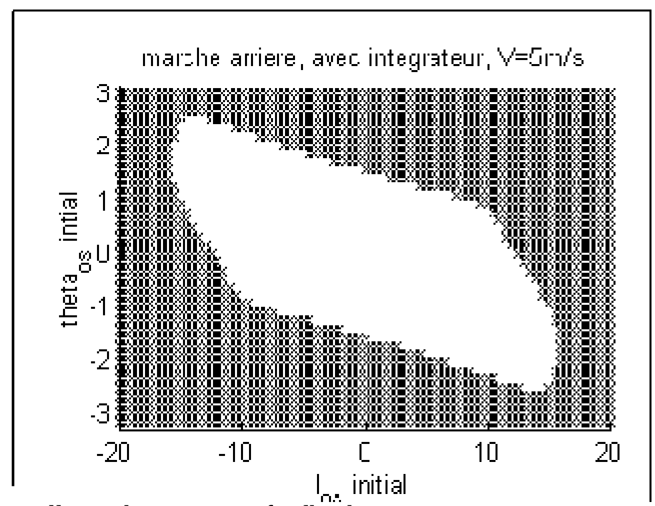


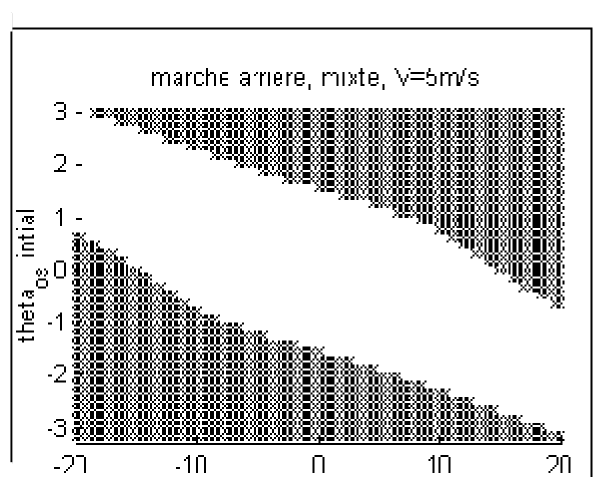
Figure 2 : A Tractor-trailer with positive off-axle hitching



Controller with static state-feedback



Controller with static state-feedback plus integral action



Hybrid controller

Figure 3: Region of convergence for the various controllers' configurations