Further experimental results on nonlinear control of flexible joint

# manipulators

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## Abstract

In this note we focus on the experimental comparison of feedback controllers for flexible joint manipulators. It completes a previous set of experimental results presented in [3], on a 2 degree-of-freedom (dof) manipulator with a stiff second joint (50 Nm/rad). The process considered here is quite different since it consists of a 1 dof flexible joint manipulator (i.e. a 4<sup>th</sup> order linear system) with a high flexibility (3.4 Nm/rad). The main goal of the present work is to apply a systematic way of tuning the gains of backstepping- and energy shaping-like controllers, using a method developed for LQ controllers by De Larminat [1]. Notice that although other linear controllers could be applied to the process tested in this note. we restrict ourselves to those control laws that extend to the nonlinear general case of flexible joint manipulators, and yield global stability results.

#### **1** Introduction

In this note we continue the work started in [4] [5] and [3] that concerns the control of *n*-dof flexible joint manipulators, using the static state feedback linearizable model introduced by Spong [2]. The process to be controlled is depicted in figure 1 where  $I_1 = 0.0085$  kg.m<sup>2</sup> and  $I_2 = 0.0078$  kg.m<sup>2</sup> are the inertias, k = 3.4 Nm/rad is the equivalent angular stiffness. Those values have

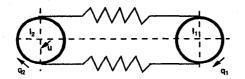


Figure 1: The 1-dof flexible joint manipulator.

been estimated off-line and can be considered as accurate. The open-loop transfer function between  $q_1$  and u has cut-off frequency  $\omega_c(OL) \approx 33 \text{ rad/s}$ . The controllers that are tested and compared in this work are PD:  $u = -k_p(q_2 - q_d) - k_v q_2$ <sup>(1)</sup>, singular perturbation based SPB

[2]:  $u = (I_1 + I_2)(\ddot{q}_d - \lambda \ddot{q}_1) - k_d(\ddot{q}_1 + \lambda \ddot{q}_1) + k_{d44}(\dot{q}_1 - \dot{q}_2) + k_{d55}(q_1 - q_2)$ , two backstepping based BACK1 [5]:  $u = I_2[\ddot{q}_{2d} - (k_1 + k_2)\ddot{q}_2 - (1 + k_1k_2)\ddot{q}_2 - k(\dot{s}_1 + s_1)] + k(q_2 - q_1)$ , BACK2 [5]:  $u = I_2[\ddot{q}_{2d} - (k_1 + k_2)\dot{q}_2 - (1 + k_1k_2)\ddot{q}_2 - (\dot{s}_1 + s_1)] + k(q_2 - q_1)$ , and one passivity based MES [5] [4]:  $u = I_2[\ddot{q}_2 - \lambda \dot{q}_2] + k(q_{2d} - q_{1d}) - k_{d33}s_2$ , see [5] [3] for further details. Those schemes require only measurement of positions and velocities. Notice that the only difference between the two backstepping schemes BACK1 and BACK2 is the last term between brackets. In [5] it was pointed out that this could yield high gain in the closed-loop, and that was confirmed experimentally in [3] where the first backstepping scheme BACK1 was impossible to implement.  $q_d(t) = \sin(\omega t)$  or  $q_d(t) = \frac{b^5}{(s+b)^5}[g(t)]$  with g(t) a square signal.

### 2 Experimental results

The tuning method: Actually one should notice that despite the fact that the nonlinear backstepping and energy shaping controllers have a linear structure when applied to a linear system, their gains appear in a very nonlinear way in the state feedback matrix. As an example, the term multiplying  $q_1$  for the MES scheme is equal to  $-(\lambda k_{d33}+k)\frac{k_{d11}\lambda}{k}+(k_{d33}+I_2\lambda)\frac{\lambda I_1+k_{d11}}{I_1}+I_2\frac{k_{d11}\lambda}{I_1}$ . The tuning method used in this note bases on the method proposed in [1] that applies to LQ controllers: it allows one to choose the weighting matrices of the quadratic form to be minimized, in accordance with the desired closed-loop bandwidth (or cut-off frequency  $\omega_c(CL)$ ). Therefore one gets an "optimal" state feedback matrix FLQ, with a controller u = FLQx in the case of regulation. Since the 5 tested controllers yield some state feedback matrices FPD, FSPB, FBACK1, FBACK2 and FMES respectively, which are (highly) nonlinear functions of the gains as shown above, we choose to calculate their gains (i.e.  $\lambda, k_{d33}, k_{d44}$ ...) so that the norms ||FLQ| – Fcontroller || are minimum. This amounts to solving a nonlinear set of equations f(G) = 0 where G is the vector of gains. This is in general a hard task, since we do not a priori know any root (otherwise the job would be done). This has been done numerically by constructing a grid in the gain space of each scheme and minimizing the above norm with a standard optimization routine. In 2209

<sup>&</sup>lt;sup>1</sup>Experiments show that replacing  $-k_v \dot{q}_2$  by  $-k_v \ddot{q}_2$  does not improve the closed-loop behaviour.

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figure 7,  $e = \int_{10}^{20} \tilde{q}_1(t)^2 dt$ .

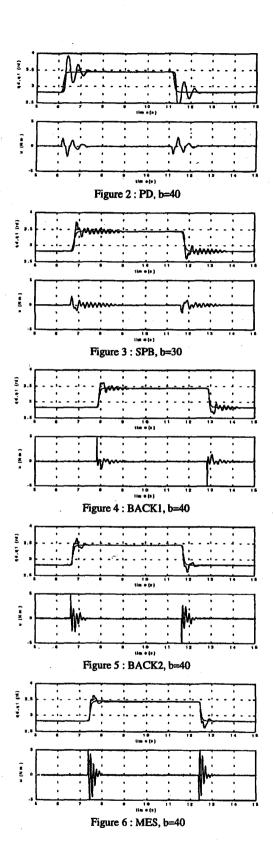
Conclusions: The major conclusions of [3] and of the present note are that i) taking joint flexibility into account at the control design stage is important to reduce significantly tracking errors, see e.g. figures 2,8 and 10-15, 4-6 ii) PD controllers yield acceptable results only for slow enough trajectories and high enough stiffnesses, otherwise the transient becomes too violent (our system strikes hard limits and the control is desactivated), iii) despite of crude link velocity estimation via numerical differentiation of a potentiometer position signal, nonlinear state feedback controllers behave correctly, see figures 10-15 and 3-6 iv) the actuators and current drivers neglected dynamics may have a significant influence on the closed-loop behaviour: we noticed experimentally a resonance phenomenon in the closed-loop (see figure 7), which was confirmed numerically by replacing u with  $u_f = \frac{u}{1+r_s}$  (<sup>2</sup>), v) backstepping and passivity (or energy shaping) controllers may yield similar closedloop behaviours, but the latter yield a "one-shot" design, vi) the systematic gain tuning method inspired from [1] provides quite acceptable set of gains in the sense that several controllers behave correctly despite of fast desired trajectories, see e.g. figures 13,15,5,6, vii) the SPB controller has lower performance than the BACK2 and MES ones, compare figures 3 and 5.6 viii) the BACK1 controller performs well but still its performance is less good than those of BACK2 and MES, see figures 11,4 and 13,15.

Future work will mainly concern the extension of the systematic tuning method to nonlinear manipulators as in [3].

### References

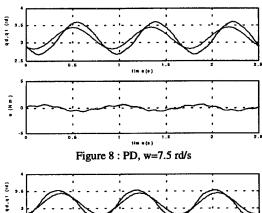
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<sup>&</sup>lt;sup>2</sup>In fact iii and iv show that developing velocity observers for such systems may not be so important, whereas some neglected dynamics, whose influence has received less attention in the literature, have a significant effect.



u	PD		SPB		BACK1		BACK2		MES	
w(rd:s)	е	(q1-q1d)max	e	(q1-q1d)max	e	(q1-q1d)max	e	(q1-q1d)max	e	(q1-q1d)max
2.5	0.70	0.0630	0.20	0.0385	0.21	0.0293	0.25	0.0374	0.33	0.0386
5	3.54	0.0943	2.14	0.0877	2.57	0.1138	1.54	0.0840	2.78	0.0983
7.5	20.86	0.1946	10.14	0.1851	8.53	0.1501	4.17	0.1040	7.92	0.1472
10					20.60	0.2428	13.00	0.1823	19.03	0.2150
12.5					48.07	0.4138	35.15	0.2965	36.05	0.2910
15					63.44	0.4494	53.33	0.3418	31.03	0.2581
20					37.70	0.2842	2.97	0.0842	8.58	0.1364

Figure 7 : Comparison tableau



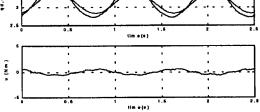
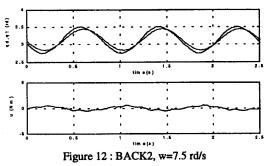


Figure 10 : BACK1, w=7.5 rd/s



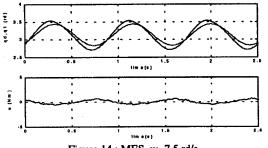


Figure 14 : MES, w=7.5 rd/s

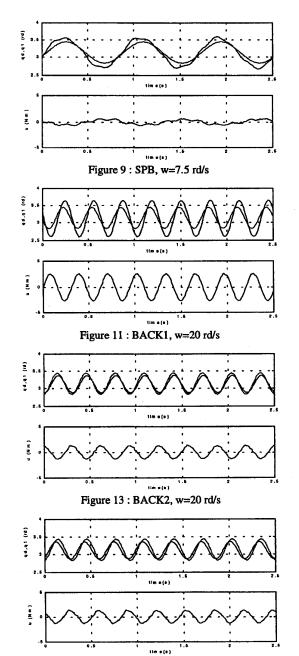


Figure 15 : MES, w=20 rd/s