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Modeling, stability and control of biped robots—a general framework☆

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Abstract

The focus of this survey is the modeling and control of bipedal locomotion systems. More specifically, we seek to review the developments in the field within the framework of stability and control of systems subject to unilateral constraints. We place particular emphasis on three main issues that, in our view, form the underlying theory in the study of bipedal locomotion systems. Impact of the lower limbs with the walking surface and its effect on the walking dynamics was considered first. The key issue of multiple impacts is reviewed in detail. Next, we consider the dynamic stability of bipedal gait. We review the use of discrete maps in studying the stability of the closed orbits that represent the dynamics of a biped, which can be characterized as a hybrid system. Last, we consider the control schemes that have been used in regulating the motion of bipedal systems. We present an overview of the existing work and seek to identify the needed future developments. Due to the very large number of publications in the field, we made the choice to mainly focus on journal papers.

Keywords: Biped robots; Non-smooth mechanics; Unilateral constraints; Complementarity conditions; Multiple impact laws; Hybrid system; Gait stability; Control synthesis

1. Introduction

In general, a bipedal locomotion system consists of several members that are interconnected with actuated joints. In essence, a man-made walking robot is nothing more than a robotic manipulator with a detachable and moving base. Design of bipedal robots has been largely influenced by the most sophisticated and versatile biped known to man, the man himself. Therefore, most of the models/machines developed bear a strong resemblance to the human body. Almost any model or machine can be characterized as having two lower limbs that are connected through a central member. Although the complexity of the system depends on the number of degrees of freedom, the existence of feet structures, upper limbs, etc., it is widely known that even extremely simple unactuated systems can generate ambulatory motion. A bipedal locomotion system can have a very simple structure with three point masses connected with massless links (Garcia, Chatterjee, Ruina, & Coleman, 1997) or very complex structure that mimics the human body (Vukobratovic, Borovac, Surla, & Stokic, 1990). In both cases, the system can walk several steps. The robotics community has been involved in the field of modeling and control of bipeds for many years. The books (Vukobratovic, 1976; Vukobratovic et al., 1990; Raibert, 1986; Todd, 1985) are worth reading as an introduction to the field. The interested reader may also refer to the following web pages:

http://www.androidworld.com/prod28.htm,
http://robby.caltech.edu/~kajita/bipedsite.html,
http://www.fzi.de/divisions/ipt/WMC/preface/preface.html,
http://www.kimura.is.uec.ac.jp/faculties/legged-robots.html.

Nevertheless, and despite the technological exploit achieved by Honda's engineers (Japan is certainly the country where bipedal locomotion has received the most attention and has the longest history), some fundamental

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Manfred Morari Editor. In this survey, we review research efforts in developing control algorithms to regulate the dynamics of bipedal gait. We focus on issues that are related to modeling, stability, and control of two legged locomotion systems.

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modeling and control problems have still not been addressed nor solved in the related literature. One may notice, in particular, that the locomotion of Honda's P3 prototype remains far from classical human walking patterns at the same speeds. Although Honda (HONDA) did not publish many details either on the mechanical part or on the implemented control heuristic, it is easy to see on the available videos that P3's foot strike does not look natural and leads to some transient instability (http://www.honda-p3.com). The number of foot design patents taken out by Honda (up to an air-bag-like planter arch) reveals again that foot–ground impact remains one of the main difficulties one has to face in the design of robust control laws for walking robots. This will become the key issue with increasing horizontal velocity requirement. This problem, however, is more sensitive for two-legged robots than for multi-legged ones due to the almost straight leg configuration and the bigger load at impact time for the former, leading to stronger velocity jumps of the center of mass. While Honda's engineers seem to consider these velocity jumps as unwanted perturbations and thus appeal to mechanical astuteness to smooth the trajectory, we argue that impact is an intrinsic feature of mechanical systems like biped robots and should be taken as such in the controller design. Other bipedal robots have been designed. Among the most advanced projects, we cite the Waseda University Humanoid Robotics Institute biped, the MIT Leg Laboratory robots, the LMS-INRIA BIP system (Sardain, Rostami, & Bessonnet, 1998; Sardain, Rostami, Thomas, & Bessonnet, 1999), the CNRS-Rabbit project (Chevallereau et al., 2003), and the German Autonomous Walking programme (Gienger, Löffler, & Pfeiffer, 2003), which can be found at

http://www.humanoid.rise.waseda.ac.jp/booklet/kato_4.html,
http://www.ai.mit.edu/projects/leglab/robots/robots.html,
http://www.inrialpes.fr/bip,
http://www-lag.ensieg.inpg.fr/PRC-Bipedes/,
http://www.fzi.de/ids/dfg_schwerpunkt_laufen/start_page.html.

respectively. Among all these existing bipeds, the Honda robots seem to be the most advanced at the time of writing of this paper according to the information made available by the owners. However, the solution for control designed by Honda does not explain why a given trajectory works nor does it give any insight as to how to select, chain together, and blend various behaviors to effect locomotion through difficult terrain (Pratt, 2000). It is the feeling of the authors that the problem of feedback control of bipedal robots will not be solved properly as long as the dynamics of such systems is not thoroughly understood. In fact, the main motivation for the writing of this paper has been the following observation about walking: there is no analytical study of a stable controller with a complete stability proof available in the related literature. It is our belief that the main reason for this is the lack of a suitable model. We propose a framework that is not only simple enough to allow subsequent stability and control studies but also realistic as some experimental validations prove. In addition, the framework provides a unified modeling approach for mathematical, numerical, and control problems, which has been missing. It is for instance significant that the main efforts of the MIT Leg Lab (Pratt, 2000) have been directed toward technological (actuators) improvement and testing of heuristic control algorithms similar to Honda's works.

We should emphasize that the main thrust of this survey does overlook several practical aspects that may arise during the design and development of walking machines. Admittedly, a walking machine can be built without paying attention to many of the main ideas of this survey. There are numerous toys that walk in a certain fashion. There are quite a few bipedal robots that are designed to avoid impacts altogether during walking. The fact remains that the stability, agility, and versatility of any existing bipedal machine does not even come close to that of the human biped. The surveyed concepts will better enable the design and evaluation of such machines through more suitable control algorithms that take into account impact mechanics and stability. The practical issues that arise in the design and development actual machines deserve another survey article. In the ensuing part of this survey we, therefore, will mainly focus on a theoretical framework.

2. General description of a bipedal walker

A biped can be represented by an inverted pendulum system that has a constrained motion due to the forward and backward impacts of the swing limb with the ground (Cavagna, Heglund, & Taylor, 1977; Hurmuzlu & Moskowitz, 1986; Full & Koditschek, 1999). Although similar to the structure of vibration dampers in many aspects (Shaw & Shaw, 1989), which are relatively well studied, structure of bipedal systems have a fundamental difference arising from the unconstrained contact of the limbs with the ground (see Fig. 1). While the vibration damper remains in contact with the reference frame at all times because of the hinge that is located between the inverted pendulum and the vibrating mass, the limbs of the biped are always free to detach from the walking surface. Detachments occur frequently and lead to various types of motion such as walking, running, jumping, etc. As a matter of fact, one can classify bipedal locomotion systems as complementarity systems (Lötstedt, 1984; Brogliato, 2003). Such a modeling framework does not at all preclude the introduction of flexibilities at the contact points. Also it allows one to include other effects like Coulomb friction in a single framework, which can be quite useful for numerical simulations in order to validate the controllers. Fig. 1 depicts other systems that fall into the same category. In the latter part of the article, we will show that such systems can be analyzed by the use of Poincaré maps (Hurmuzlu & Moskowitz, 1987), and possess common features in terms of motion control. We would like to stress that the focus of this article is on motions that include

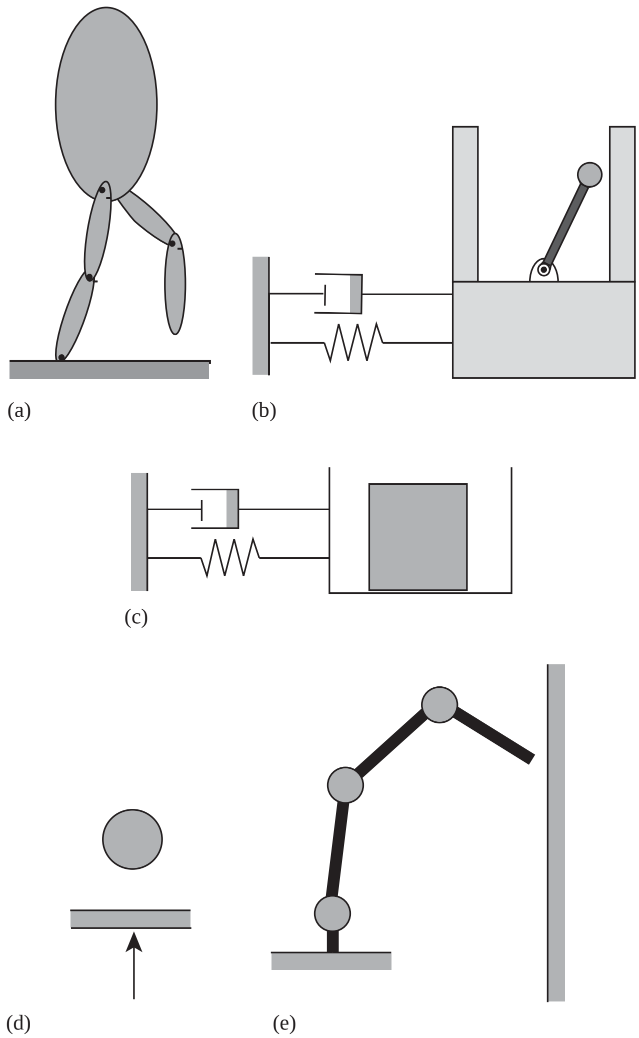

Fig. 1. Bipeds as complementarity systems: (a) biped, (b) pendulum, impact damper, (c) mass impact damper, (d) juggler, and (e) manipulator in contact with a rigid wall.

contact and impacts. For example, a biped can rock back and forth while the swing limb remains above the walking surface for all times. Such motions will be outside the focus of the discussions presented here.

A typical walking cycle may include two phases: the single support phase, when one limb is pivoted to the ground while the other is swinging in the forward direction (open kinematic chain configuration), and the double support phase, when both limbs remain in contact with the ground while the entire system is swinging in the forward direction (closed kinematic chain configuration). When both limbs are detached, the biped is in the "flight" phase and the resulting motion is running or some other type of non-walking motion (Kar, Kurien, & Jayarajan, 2003). Any effort that involves analytical study of the dynamics of gait necessitates a thorough knowledge of the internal structure of the locomotion system. When the system is human or animal, this structure is extremely complicated and little is

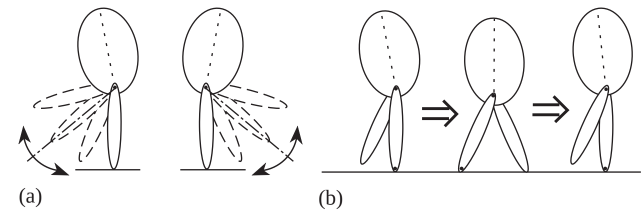

Fig. 2. (a,b) Types of oscillation of a three-element biped.

known about the control strategy that is used by human beings and animals to realize a particular motion and achieve stable gait (Full & Koditschek, 1999; Vaughan, 2003). If the system represents a man-made machine or a numerical model representing such a machine, one has to synthesize control strategies and performance criteria that transform multi-body systems to walking automata. Devising practical control architectures for bipedal robots remains to be a challenging problem. The problem is tightly coupled with the control studies in the area of robotic manipulators. Unlike manipulators, however, bipedal machines can have many types of motion. The control objectives should be carefully selected to conform with a specific type of motion. A control strategy that is selected for high-speed walking may cause the system to transfer to running, during which an entirely different control strategy should be used, similar to jugglers control (Brogliato & Zavala-Rio, 2000).

Consider the biped depicted during the single support phase in Fig. 1(a). This biped is equivalent to a pendulum attached to the foot contact point with a mass and a length that are configuration dependent. Inverted pendulum models of various complexities, therefore, have been extensively used in the modeling of gait of humans and bipedal walking machines. The dynamics of bipedal locomotion is intuitively similar to that of an inverted pendulum and has been shown to be close to it according to energetical criteria (Kar et al., 2003; Cavagna et al., 1977; Full & Koditschek, 1999; Blickman & Full, 1987).

A simple two-element biped (Hurmuzlu & Moskowitz, 1987) may operate in two modes (see Fig. 2): (a) impactless oscillations, (b) progression with ground contacts. The importance of the contact event can be better understood if the motion is depicted in the phase space of the state variables.

We simplify the present discussion by describing the events that lead to stable progression of a biped for a single-degree-of-freedom system, however, this approach can be generalized to higher-order models. The phase plane portrait corresponding to the previously described dynamic behavior is depicted in Fig. 3(a). The sample trajectories corresponding to each mode of behavior are labelled accordingly. The vertical dashed lines represent the values of the coordinate depicted in the phase plane for which the contact occurs. For the motions depicted in Fig. 3(a) or (b), the only trajectory that leads to contact is C. The contact event for this simple model produces two

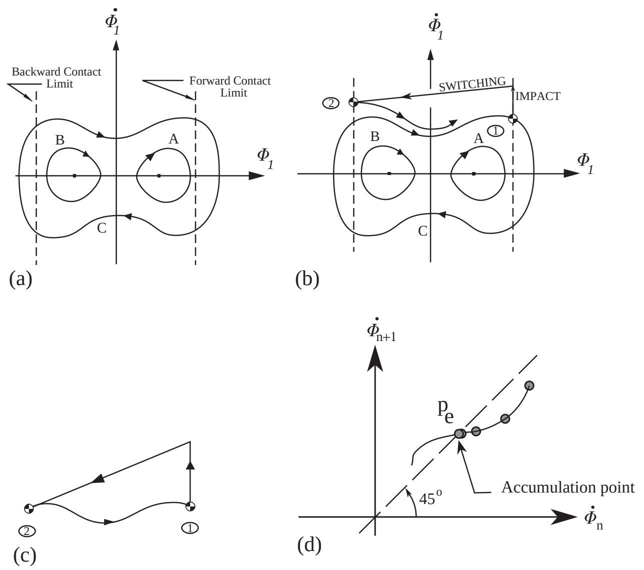

Fig. 3. Impact on the phase plane portrait.

simultaneous events:

(1) impact, which is represented by a sudden change in generalized velocities,
(2) switching due to the transfer of pivot to the point of contact.

The combined effect of impact and switching on the phase plane portrait is depicted in Fig. 3(b). As shown in the figure, the effect of the contact event will be a sudden transfer in the phase from points 1 to 2, which is generally located on a different dynamic trajectory than the original one. If the destination of this transfer is on the original trajectory, then the resulting motion becomes periodic (Fig. 3(c)). This type of periodicity has unique advantage when the inverted pendulum system represents a biped. Actually, this is the only mode of behavior that this biped can achieve progression. The most striking aspect of this particular mode of behavior is that the biped achieves periodicity by utilizing only a portion of a dynamic trajectory. The impact and switching modes provide the connection between the cyclic motions of the kinematic chain and the walking action.

We can clearly observe from the preceding discussion that the motion of a biped involves continuous phases separated by abrupt changes resulting from impact of the feet with the walking surface. During the continuous phase, we may have none, one, or two feet in simultaneous contact with the ground. In the case of one or more feet contacts, the biped is a dynamical system that is subject to unilateral constraints. When a foot impacts the ground surface, we face the impact problem of a multi-link chain with unilateral constraints. In fact, the overall motion of the biped may include a very complex sequence of continuous and discontinuous phases. This poses a very challenging control problem, with an added complication of continuously changing motion constraints and large velocity perturbations resulting from ground impacts.

3. Mathematical description of a biped as a system subject to unilateral constraints

3.1. Dynamics of the complementarity model

Bipedal locomotion systems are *unilaterally constrained* dynamical systems. A way to model such systems is to introduce a set of unilateral constraints in the following form:

$$F(q) \geqslant 0, \quad q \in \mathbb{R}^p, \quad F : \mathbb{R}^p \to \mathbb{R}^m,$$

where q represents the complete vector of independent generalized coordinates. In other words, p denotes the number of degrees of freedom of the system without constraints, i.e. when $F(q) > 0$. The constraints mean that the bodies that constitute the system cannot interpenetrate (irrespective of the fact that they are rigid or flexible). The dynamics of a p-degree-of-freedom mechanical system subject to m unilateral constraints may be written as the following system $(\mathscr{S})$ of equations, named a complementarity dynamical system (Brogliato, 2003; Heemels & Brogliato, 2003):

$$M(q)\ddot{q} + N(q,\dot{q}) = Tu + \nabla F(q)\lambda_n + P_t(q,\dot{q}), \tag{1}$$

$$\lambda_n^{\mathrm{T}} F(q) = 0, \quad \lambda_n \geqslant 0, \quad F(q) \geqslant 0, \tag{2}$$

Restitution law + shock dynamics, (3)

Dry Friction Amontons–Coulomb′s model, (4)

where $M(q)$ is the inertia matrix, $N(q,\dot{q})$ includes Coriolis, centrifugal, gravitational, and other terms, u is an external input, $\lambda_n \in \mathbb{R}^m$ is the Lagrange multiplier corresponding to the normal contact force. The orthogonality $\lambda_n^{\mathrm{T}} F(q) = 0$ means that if $F(q) > 0$ then $\lambda_n = 0$, whereas a non-zero contact force $\lambda_n > 0$ is possible only if there is contact $F(q) = 0$. Such a contact model therefore excludes gluing, magnetic forces. Complementarity Lagrangian systems as in (1)–(4) have been introduced by Moreau (1963, 1966), and Moreau and Panagiotopoulos (1988). The rest of the terms and variables are defined next. For the bipeds the Lagrange dynamics can be rewritten in a specific way that corresponds to control objectives and allows the designer to get a better understanding of their dynamical features (Wieber, 2000; Grizzle, Abba, & Plestan, 2001; Werstervelt, Grizzle, & Koditschek, 2003). In other words, the choice of the generalized coordinates q is crucial for control purpose, and certainly much less obvious than it is for serial manipulators. For example in Fig. 4, q can be split in two subsets q_1 and q_2. The vector $q_1 = (x, y, \theta)$ describes the global position of the robot in space whereas the vector $q_2 = (\alpha_1, \ldots, \alpha_6)$ encapsulates the joint coordinates. The vector q_1 could be attached to any point of the biped. Nevertheless, it is known from biomedical studies of human gait that one of the primary objectives

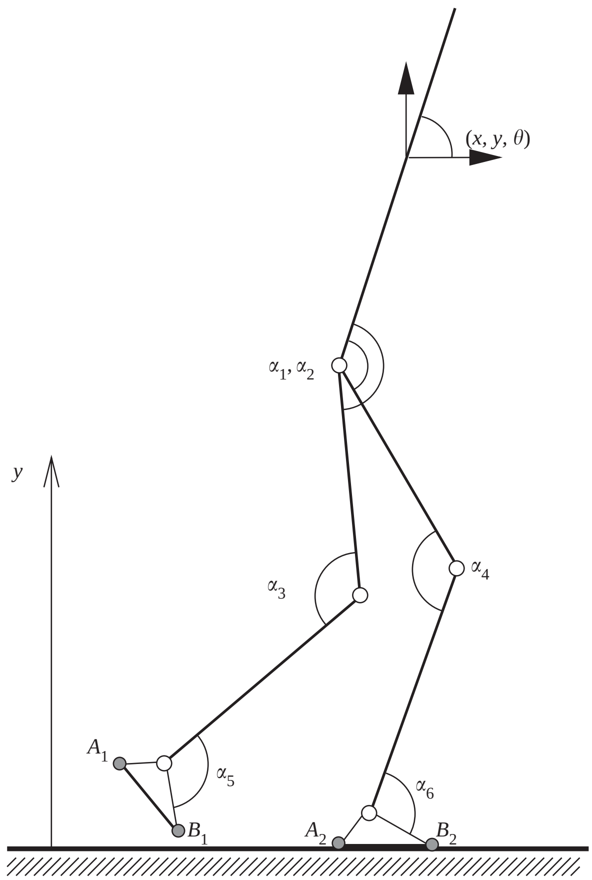

Fig. 4. A 9-degree-of-freedom planar biped.

during locomotion is the stabilization of the head where the exteroceptive sensors (inner ear, sight) are located. Setting $q=(q_1,q_2)^{\mathrm{T}}$, (1) splits in an upper part and a lower part corresponding to head motion and joint dynamics, respectively,

$$\begin{pmatrix} M_1(q) \\ M_2(q) \end{pmatrix} \ddot{q} + \begin{pmatrix} N_1(q,\dot{q}) \\ N_2(q,\dot{q}) \end{pmatrix} = \begin{pmatrix} 0 \\ T_2 \end{pmatrix} u + \nabla F(q)\lambda_n + P_t(q,\dot{q}), \tag{5}$$

$$F(q) = (y_{A_1}, y_{B_1}, y_{A_2}, y_{B_2})^{\mathrm{T}} \in \mathbb{R}^4, \tag{6}$$

where y_{A_1} denotes the y-coordinate of point A_1, and so on. In the sequel we will review the available modeling tools which will allow the designer to complete (5) with (3) and (4). In (5), $T_1=0$ since the biped has only joint actuators. As a matter of fact head motion can only be achieved thanks to a coordinated action of joint actuation and contact forces. This fact is probably more apparent for the flight phases (running) during which there are no contact forces and the trajectory of the center of mass (attaching this time q_1 to the center of mass) yields only to gravity. All complementarity systems depicted in Fig. 1 can be represented in the form given by (1)–(4). However, biped dynamics possesses some specific features that make the control study differ significantly from the control of other complementarity systems. Bipeds share the following features:

(1) *with systems* (b), (d) *and* (e): the center of mass is not controlled when $F(q)>0$, as the object of a juggler (Brogliato & Zavala-Rio, 2000),
(2) *with system* (e): their dynamics is that of a manipulator when one foot sticks to the ground,
(3) *with system* (b): they may be underactuated (no ankle actuator) and act as an inverted pendulum when one foot sticks to the ground.

The fact that bipeds merge all these characteristics makes their control analysis complex. Eq. (1) represents the dynamics when either the system evolves in free-motion or in a phase of permanent contact, i.e.

$$\begin{cases} F_i(q) \equiv 0 & \text{for some } i \in \mathscr{I}(q) \subseteq \{1,\ldots,m\}, \\ F_j(q) > 0 & \text{for } j \notin \mathscr{I}(q). \end{cases} \tag{7}$$

Then, in Eq. (1) ($\nabla F_i(q) = (\partial F_i^{\mathrm{T}}/\partial q)(q) \in \mathbb{R}^p$ is the gradient vector)

$$\nabla F(q)\lambda_n = \sum_{i=1}^{i=m} \nabla F_i(q)\lambda_{n,i}.$$

Notice that for $i \in \mathscr{I}(q)$, one can express (2) as (where $\dot{F}_i(q,\dot{q}) = \mathrm{d}/\mathrm{d}t[F_i(q(t))]$)

$$\lambda_{n,i}\dot{F}_i(q,\dot{q}) = 0, \quad \lambda_{n,i} \geqslant 0, \quad \dot{F}_i(q,\dot{q}) \geqslant 0 \tag{8}$$

or, if $\dot{F}_i(q,\dot{q}) \equiv 0$ for $i \in \mathscr{I}(q)$, as (where $\ddot{F}_i(q,\dot{q},\ddot{q}) = (\mathrm{d}/\mathrm{d}t)[\dot{F}_i(q(t),\dot{q}(t))]$)

$$\lambda_{n,i}\ddot{F}_i(q,\dot{q},\ddot{q}) = 0, \quad \lambda_{n,i} \geqslant 0, \quad \ddot{F}_i(q,\dot{q},\ddot{q}) \geqslant 0. \tag{9}$$

As we shall see, when the constraints are frictionless the conditions in (9) define a linear complementarity problem (LCP) with unknowns $\lambda_{n,i}$. An LCP is a system of the form $Ax+B \geqslant 0$, $x \geqslant 0$, $x^{\mathrm{T}}(Ax+B)=0$ (Cottle, Pang, & Stone, 1992). LCPs are ubiquitous in many engineering applications (Ferris & Pang, 1997), and particularly in unilateral mechanics in which they have been introduced by Moreau (1963, 1966). Finally, the models in (3) and (4) are needed to complete the dynamics. In particular, it is necessary to relate the post-impact velocities to the pre-impact data to be able to compute solutions that are compatible with the constraints (integrate the system and render the domain $\Phi \triangleq \{q \mid F(q) \geqslant 0\}$ invariant). The classical Coulomb friction model in (4) provides the form of $P_t(q,\dot{q})$ in (1). It is apparent that the set of equations in (1)–(9) defines a complex hybrid dynamical system, in the sense that it mixes both continuous and discrete-event phenomena. The states of the discrete-event system (DES) are defined by the 2^m modes of the complementarity conditions in (2). It is also important to point out that such systems fundamentally differ in nature from those studied in Bainov and Simeonov (1989), which consist of mainly ordinary differential equations with impulsive disturbances. Some discrepancies between both models are recalled in Brogliato (1999, Sections 1.4.2, 7.1) and Brogliato, ten Dam, Paoli, Génot, and Abadie (2002). Especially, one should always keep in mind that the complementarity conditions (2) play a major role in the dynamics of complementarity systems (for instance they are necessary to properly characterize the equilibrium states). From a mechanical engineer point of view, two questions have to be answered to, when one wants to integrate system (1)–(4) (Moreau & Panagiotopoulos, 1988):

Q1. Assume that $\mathscr{I}(q)$ in (7) is non-empty at a time instant τ, and that the velocity $\dot{q}(\tau^-)$ points inwards Φ or tangentially to $\partial\Phi$: determine which contacts $i \in \mathscr{I}(q)$, will persist at τ^+. In other words, determine the subsequent mode (or DES state).

Q2. At an impact time t_k, one has $\dot{q}(t_k^-)^{\mathrm{T}}\nabla F_i(q(t_k)) < 0$ for some $i \in \{1,\dots,m\}$ and $F_i(q(t_k))=0$. Determine the right velocity $\dot{q}(t_k^+)$. In other words, determine a re-initialization of the (continuous) state as soon as one (or several) of the unilateral conditions in (2) is going to be violated.

In the sequel t_k, $k=0,1,2\dots$ will generically denote the impact instants. The answer to those questions is far from being trivial and has been the object of many researches. The first one is related to solving the LCP associated to the system, i.e. being able to calculate at each time of a collision-free phase the interaction forces $P_q \triangleq \nabla F(q)\lambda_n + P_t(q,\dot{q})$; hence, the acceleration $\ddot{q}$ and the subsequent motion. It has been raised initially by Delassus (1917) (see also Pfeiffer & Glocker, (1996) for a very nice and simpler example). The second question is that of defining proper restitution rules, or collision laws. This goes back to the 17th century and the celebrated Newton's conjecture, see Brogliato (1999) and Kozlov and Treshchev (1991) for more details. In particular, if several hypersurfaces $\Sigma_i = \{q \mid F_i(q)=0\}$ are attained simultaneously, a *multiple impact* occurs. Such an event occurs typically during walking at the end of a single support phase when the swinging leg foot hits the ground. For example, for the planar biped depicted in Fig. 4, a 3-impact takes place at foot strike, that is $y_{A_2}(t)=y_{B_2}(t)=0$, $y_{A_1}(t)>0$, and $y_{B_1}(t)>0$ for $t<t_k$, while $y_{A_2}(t_k)=y_{B_2}(t_k)=0$, $y_{A_1}(t_k)=0$, and $\dot{y}_{A_1}(t_k^-)<0$.

From our point of view, only too few attention was payed in the literature to the existence of unilateral constraints. In fact, most of the works on the control of biped robots model the robot in the single support phase as a manipulator whose base corresponds to the supporting foot and add some closed loop constraints for the double support phase. Even if this approach is very convenient to derive trajectory tracking laws via the computed torque technique, it does not account for possible slippage nor detachment at ground contacts nor say anything about the influence of impacts on stability. Notice also that the human walking pattern and the underlying control strategy differs notably depending on the ground characteristics (ice arena, basketball playground, trampoline, etc.), which makes the walking pattern (or trajectory planning) an important topic of research (El Hafi & Gorce, 1999; Lum, Zribi, & Soh, 1999; Yagi & Lumelsky, 2000; Rostami & Bessonnet, 2001; Shih, 1999; Huang et al., 2001; Chevallereau & Aoustin, 2001; Saidouni & Bessonnet, 2003).

3.2. Frictionless contacts (continuous motion)

The succeeding sections are devoted to the study of the LCP in (9) (i.e. the calculation of the contact forces) and multiple impacts respectively. In the frictionless case ($\mathscr{S}$) reduces to (1), (2) and (3) with $P_t(q,\dot{q})=0$. Moreau (1963, 1966) has been the first to show that in the multiple constraints case $m \geq 1$, using that $\ddot{F}(q,\dot{q},\ddot{q}) = \nabla F^{\mathrm{T}}(q)\ddot{q} + f(q,\dot{q})$, the complementarity relation (9) combined with (1) yields an LCP of the form (assuming here that $\mathscr{I}(q) = \{1,\dots,m\}$ in (7))

$$\ddot{F}(q,\dot{q},\ddot{q}) = A(q)\lambda_n + b(q,\dot{q}) \geqslant 0,$$
$$\lambda_n \geqslant 0 \quad \text{and} \quad \lambda_n^{\mathrm{T}}\ddot{F}(q,\dot{q},\ddot{q}) = 0, \tag{10}$$

where

$$A(q) = \nabla F^{\mathrm{T}}(q)M^{-1}(q)\nabla F(q)$$

and

$$b(q,\dot{q}) = \nabla F^{\mathrm{T}}(q)M^{-1}(q)h(q,\dot{q}) + f(q,\dot{q})$$

and setting $h(q,\dot{q}) = Tu - N(q,\dot{q})$. If the active constraints ($i \in \mathscr{I}(q)$) are independent, then A is positive symmetric definite (PSD) and it is known that the LCP in (10) possesses a unique solution λ_n (Cottle et al., 1992). If some constraints are dependent, then A is only semi-PSD and uniqueness only holds for the acceleration $\ddot{F}(q,\dot{q},\ddot{q})$. Moreover, $\nabla F(q)\lambda_n$ is also unique, see Moreau (1966) and Lötstedt (1982), and thus from dynamics (1) $\ddot{q}$ is unique too. As a classical example, one may think of a chair with four legs on a rigid ground: even if the interaction forces cannot be uniquely determined, the acceleration of the mass center is unique (upwards). Using Kuhn–Tucker's theory (Kuhn & Tucker, 1951) it is possible to show that any solution λ_n to the LCP in (10) is also a solution to the quadratic problem

$$\min_{\lambda_n \geqslant 0} \tfrac{1}{2}\lambda_n^{\mathrm{T}}A(q)\lambda_n + \lambda_n^{\mathrm{T}}b(q,\dot{q}), \tag{11}$$

which is equivalent to Gauss' principle of least deviation (Moreau, 1966; Lötstedt, 1982). To summarize, it is clear that if one is able to obtain at time τ a value for λ_n, then introducing this value into (2) allows one to determine which contacts persist and which ones are going to break (become inactive) on $(\tau, \tau+\varepsilon)$, $\varepsilon > 0$, ε small enough.

Remark 1. The answer to Q1 does not necessarily require the explicit calculation of the contact forces λ_n. Let us consider the simple example ($p=1$) of a ball resting on the ground ($m=1$) at τ_0. Suppose that an external force f is applied to the ball at τ_0. Then the dynamics is $\ddot{q}(t) = \max(0, f)$ (recall that max functions can be written with complementarity). Let us discretize the unconstrained

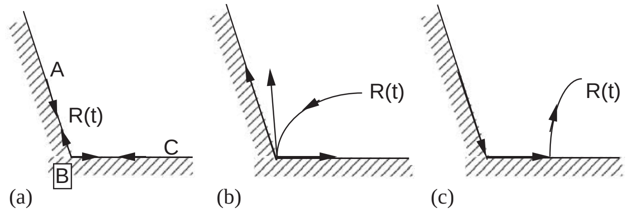

Fig. 5. Possible motions and shock outcomes in an abstract configuration space.

motion ($q > 0$) as

$$q_{i+1} = q_i + h\dot{q}_i,$$
$$\dot{q}_{i+1} = \dot{q}_i + hf. \quad (12)$$

Using (12), the dynamics is discretized as

$$\dot{q}_{i+1} = \text{prox}[\mathbb{R}^+, \dot{q}_i + hf], \quad (13)$$

where prox denotes the proximal point to $\dot{q}_i + hf$ in $\mathbb{R}^+$. This *implicit* scheme can be generalized to more complex systems ($p \geqslant 1$, $m \geqslant 1$), as proposed in Moreau (1986). Moreover, it can be generalized to provide one solution to the multiple impact problem as will be shown in Section 3.3.

3.3. Frictionless contacts (multiple impacts)

Section 3.2 was devoted to partially answer to Q1. We now focus on Q2. In general, walking robots use several support points during locomotion (biped robots; Hurmuzlu & Moskowitz, 1986, 1987; Hurmuzlu, 1993, quadruped robots, Chevallereau, Formal'sky, & Perrin, 1997; Perrin, Chevallereau, & Formal'sky, 1997). In other words $m \geqslant 2$. This means that the boundary $\partial\Phi$ of the admissible domain Φ is not differentiable everywhere. Its singularities correspond to surfaces with codimension $\geqslant 2$. Thus, the eventuality that the state collides in a neighborhood of a singularity cannot be excluded in mechanical systems as ($\mathscr{S}$). In the framework of bipedal locomotion, such events intrinsically belong to the dynamics of walking. Let us view for simplicity the bipedal robot as a point $R(t)$ in a two-dimensional generalized configuration space. The classical standard walking assumption can be seen as a "bilateral sliding motion" of the generalized point $R(t)$ confined to the two constraint boundaries, see Fig. 5(a). Single support phases correspond to $R(t) \in [A, B) \cup (B, C]$, the double support phase to $R(t) = B$. However one cannot a priori exclude

- rebounding multiple shocks at B, see Fig. 5(b);
- detachment during single support phases, see the foregoing paragraph, and Fig. 5(c).

The most recent studies in the field can be divided in two different approaches.

- The first one (Hurmuzlu & Marghitu, 1994; Marghitu & Hurmuzlu, 1995; Han & Gilmore, 1993) consists of an enumeration procedure. They apply a restitution law for simple impact at A_j and sequentially propagate its effects on the other contact points A_i, $i \in \mathscr{I}(q)$. Nevertheless, the overall process is assumed to be instantaneous, i.e. all shocks occur simultaneously at all the contact points: this is therefore really a multiple impact. At each point they investigate whether assuming zero or non-zero local percussions yield consistent outcomes. The main drawback of these studies is that they do not rule on whether or not the proposed algorithms always terminate with a unique solution. In the case of several admissible solutions, they do not give a criterion to make the choice between these solutions. However, this should not be seen as a real drawback (except for simulation tool design purpose) since any heuristic to make the "good" choice, excluding in the same time several admissible solutions, drops the rigid body assumption.
- The second approach corresponds to the definition of a *collision mapping*

$$P_c : \partial\Phi \times \{-V(q(t_k))\} \to \partial\Phi \times \{V(q(t_k))\},$$
$$(q(t_k), \dot{q}(t_k^-)) \mapsto (q(t_k), \dot{q}(t_k^+)), \quad (14)$$

where $V(q) = \{v \in \mathbb{R}^p : \forall i \in \mathscr{I}(q), v^T \nabla F_i(q) \geqslant 0\}$ denotes the tangent cone to $\partial\Phi$ at $q(t)$ (Moreau & Panagiotopoulos, 1988), i.e. the set of admissible post-impact velocities. It is clear that the choice of P_c should yield a mathematically, mechanically and numerically coherent formulation of the studied phenomenon. The so-called "sweeping or Moreau's process" (Brogliato, 1999; Moreau, 1985; Moreau & Panagiotopoulos, 1988) is a general formulation of the dynamics in (1)–(4) based on convex analysis tools. It allows one to write the dynamics as a particular *Measure Differential Inclusion*, a term coined by J.J. Moreau. It implicitly defines a collision mapping based on the computation of the post-impact motion via a proximation procedure in the kinetic metric

$$\dot{q}(t_k^+) = \text{prox}_{M(q(t_k))}[\dot{q}(t_k^-), V(q(t_k))]. \quad (15)$$

It is noteworthy that this can also be equivalently written as a quadratic programme under unilateral constraints, and consequently under a LCP formalism, see (10), (11). Let us note that the mapping in (15) applied to the shock of two rigid bodies correspond to taking a zero restitution coefficient. Hence it may be named a "generalized dissipative impact rule". However, it is possible to introduce some restitution $\delta \in [0, 1]$ by substituting $\dot{q}(t_k^+)$ in (15) by $\frac{1}{2}(1+\delta)\dot{q}(t_k^+) + \frac{1}{2}(1-\delta)\dot{q}(t_k^-)$ (Moreau & Panagiotopoulos, 1988; Mabrouk, 1998). The interested reader may have a look at Brogliato (1999, Section 5.3) for a non-mathematical introduction to this material. Similar results have been obtained in Lötstedt (1982). In Pfeiffer and Glocker (1996), an extension of Poisson's impact law is proposed that takes the form of two LCPs (see, e.g., Brogliato, 1999, Section 6.5.6 for a simple example of application of this impact mapping).

3.4. Contacts with Amontons–Coulomb's friction

In the frictionless case, we saw that the LCP (10) was well-posed since the only possible indeterminacies, resulting from dependent active constraints, do not influence the global motion of the mechanical system (uniqueness of the acceleration still holds). The problem becomes more complicated through the introduction of dry friction, an essential parameter of the legged locomotion. The next step relies on the fact that the relay characteristic of Coulomb's friction, can be represented analytically in a complementarity formalism by introducing suitable slack variables (or Lagrange multipliers). This allows one to construct a set of complementarity conditions which monitor all the transitions from sticking to slipping, and from contact to detachment. The dynamics in (1)–(4) can consequently be rewritten as (Pfeiffer & Glocker, 1996)

$$M(q)\ddot{q} = h(q,\dot{q}) + [W(q) + N_{\text{slide}}(q)]\lambda, \tag{16}$$

$$y = \mathbf{A}(q)\lambda + \mathbf{b}(q,\dot{q}),$$

$$y \geqslant 0, \quad \lambda \geqslant 0, \quad y^{\mathrm{T}}\lambda = 0 \tag{17}$$

for appropriate matrices $\mathbf{A}(\cdot)$ and $\mathbf{b}(\cdot)$, W and N_{slide}, where the components of λ are suitable slack variables. The powerfulness of the complementarity formalism clearly appears from (16), (17) in which the overall dynamics is written in a compact way: the continuous dynamics plus complementarity conditions. This perfectly fits within the general complementarity dynamical framework (Brogliato, 2003).

4. The stability framework

The most crucial problem concerning the dynamics of bipedal robots is their stability, see e.g. http://www.ercim.org/publication/Ercim_News/enw42/espiau.html. As has been explained in Sections 2 and 3, a biped is far from being a simple set of (controlled) differential equations. Moreover, the objectives of walking are quite specific. One is therefore led to first answer the question: what is a stable biped? And, consequently, what mathematical characterization of this stability can be constructed from the complementarity models? As we will see next, this is closely related to the fact that bipeds can be considered as hybrid dynamical systems, the stability of which can be attacked from various angles. The goal of this section is to present some tools which can serve for the stability analysis of models as in (2)–(5), and which are suitable for bipeds because they encapsulate their main features. Firstly, we spend some time on describing invariant sets for complementarity systems. The point of view that is put forth is that various existing, or to be investigated, stability frameworks are better understood when invariant sets are classified. Secondly, we review the so-called impact Poincaré maps, which have been used extensively in the applied mathematics and mechanical engineering literature for vibro-impact systems (Masri & Caughey, 1966; Shaw & Holmes, 1983; Shaw & Shaw, 1989). This point of view seems natural if one considers bipeds as jugglers (Brogliato & Zavala-Rio, 2000; Zavala-Rio & Brogliato, 2001). However, it presents limitations which we point out.

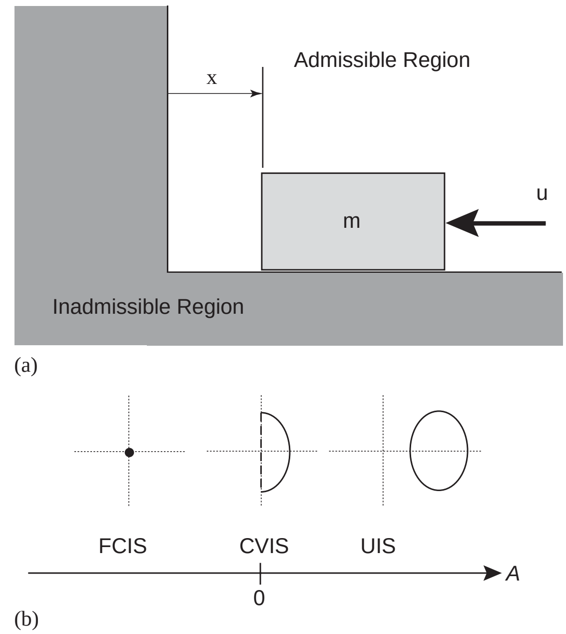

Fig. 6. Controlled mass subject to a unilateral constraint.

4.1. Invariant sets of systems subject to unilateral constraints

Application and use of mapping techniques is tightly coupled with the structure of the invariant set that represents the steady-state motion. Systems subject to unilateral constraints behave in a more complex manner than the ones that are not (Budd & Dux, 1994). For example, let us consider the system given in Fig. 6(a). Suppose we would like to develop a controller to place the mass at a time varying position $x_d(t) = A + B\sin(\omega t)$ starting from an arbitrary initial condition in the admissible region. One can use a simple controller that yields the following closed-loop dynamics:

$$m(\ddot{x} - \ddot{x}_d(t)) + k_1(\dot{x} - \dot{x}_d(t)) + k_2(x - x_d(t)) = \lambda_n,$$
$$0 \leqslant \lambda_n \perp x \geqslant 0, \quad \dot{x}(t_k^+) = -e\dot{x}(t_k^-) \text{ with } e \in [0,1). \tag{18}$$

Our objective here is not to explore all possible types of invariant sets that may be attained by the system given by (18). Instead, we would like to show that the invariant sets of the closed loop system can be classified under three categories (see Fig. 6(b)):

(i) *Constraint violating invariant sets* (*CVIS*): An invariant set that includes at least one collision with the constraint surface per cycle of motion (including orbits that stabilize in finite time on the constraint surface after an infinite number of collisions). Hence, an *impact Poincaré mapping P* is well defined that captures these orbits. This type of invariant set is a unique feature of systems subject to unilateral constraints. All the systems shown in Fig. 1 can be made to exhibit this behavior with a set of properly selected parameter values. Specifically, for locomotion systems, this will be the only mode of motion that can describe running and walking.

(ii) *Unconstrained invariant sets* (*UIS*): An invariant set that does not include collisions with the constraint surface. In the single mass case, this corresponds to the cyclic motions of the mass that occur to the right of the constraint surface ($x > 0\ \forall \bar{t} \leqslant t \leqslant \infty$). This type of invariant set can be observed for all systems of Fig. 1 except the juggler. For the biped, this corresponds to rocking when one or two limbs are in contact with the ground.

(iii) *Fully constrained invariant sets* (*FCIS*): An invariant set that never leaves the constraint surface. This corresponds for the system in (18) to a static equilibrium where the system rests on the constraint surface ($x(t) = 0\ \forall \bar{t} \leqslant t \leqslant \infty$). All the systems in Fig. 1 can exhibit this behavior with a specific choice of the control parameter values and initial conditions.

It is easy to imagine that trajectories of a system may be CVIS, FCIS and UIS simultaneously. In particular, notice that although the complementarity relations in (18), i.e. $x \geqslant 0$, $\lambda_n \geqslant 0$, $x\lambda_n = 0$, a priori define two modes $x > 0$ and $x = 0$ (hence a *bimodal* system), for control and dynamic systems analysis purpose one is led to consider those phases that correspond to CVIS as independent ones (Brogliato, Niculescu, & Monteiro-Marques, 2000; Bourgeot & Brogliato, 2003): they are described neither by the free motion nor by the constrained motion dynamics but by the whole dynamics of the hybrid system.

4.2. Bipedal robots as hybrid dynamical systems

The above classification of the invariant sets naturally leads one to consider complementarity systems as in Fig. 1 as hybrid dynamical systems whose DES states are defined from the described invariant sets. In the following, we shall generically denote the phases that correspond to CVIS as I_k and those that correspond to FCIS and/or UIS as Ω_k. With some abuse of notation, we shall denote the DES states and the corresponding time intervals in the same manner. As we shall see further subdivisions will be needed. As an example let us consider an impact damper as in Fig. 1(c) and with a sinusoidal excitation applied to the basis mass: for a proper choice of the spring-dashpot, the excitation parameters and of the initial conditions, the system possesses periodic trajectories with one impact *per* period (hence CVISs) (Masri & Caughey, 1966). Thus one has

$$\mathbb{R}^+ = I_0. \tag{19}$$

Consider now a manipulator as in Fig. 1(e) that performs a complete robotic task with a succession of free-motion and constrained motions phases, during which it is explicitly required to track desired motion and/or contact force (Bourgeot & Brogliato, 2003; Brogliato, 1999; Brogliato, Niculsecu, & Orhant, 1997; Brogliato et al., 2000; Menini & Tornambé, 2001). During the force/position control phases, the trajectories will in general be both UIS (in the tangent direction to the constraint surface) and FCIS (in the normal direction). During the free-motion phases, the trajectories are UIS. It is therefore natural to split such task into three phases Ω_{2k}, Ω_{2k+1} and I_k that correspond to UIS, FCIS and CVIS, respectively, i.e.

$$\mathbb{R}^+ = \Omega_0 \cup I_0 \cup \Omega_1 \cup \Omega_2 \cup I_1 \cup \cdots. \tag{20}$$

Consider now the biped. It is clear that in order to describe walking, running and hopping motions, one needs more than the above three types of invariant sets (Kar et al., 2003). Moreover, one needs more than the three phases proposed for the manipulator case. Indeed, as we already pointed out concerning the choice of the Poincaré section, describing for instance a walking motion involves to take care of the non-sliding conditions. Hence, one is led to differentiate contact phases (FCIS and UIS) and impact phases (CVIS and FCIS and/or UIS) that slide and those that stick. Notice moreover that this may be done independently of the presence of Amontons–Coulomb friction at the contact points: friction adds modes to the plant model, whereas our description concerns the nature of the trajectories and is directly related to stability and control objectives. But it is clear that the plant modeling will strongly influence the conditions under which those modes will be activated. Such a definition yields generally a large number of DES states. We shall define only those that are needed to describe the three mentioned types of motion:

- Ω_k^{f}: flight phases (both feet detached);
- Ω_k^{sl}: left foot sticks, right foot detached;
- Ω_k^{sr}: right foot sticks, left foot detached;
- Ω_k^{dss}: double support phase, both feet stick;
- I_k^{rs}: impacts on the right foot, left foot sticks;
- I_k^{ld}: impacts on the left foot, right foot detached.

Then we obtain the following:

$$\text{Walking}: \quad \mathbb{R}^+ = \Omega_0^{\mathrm{sl}} \cup I_0^{\mathrm{rs}} \cup \Omega_0^{\mathrm{dss}} \cup \Omega_0^{\mathrm{sr}} \cup I_0^{\mathrm{rl}} \cup \Omega_1^{\mathrm{dss}} \cup \Omega_1^{\mathrm{sl}} \cup \cdots, \tag{21a}$$

$$\text{Hopping}: \quad \mathbb{R}^+ = \Omega_0^{\mathrm{f}} \cup I_0^{\mathrm{ld}} \cup \Omega_1^{\mathrm{f}} \cup I_1^{\mathrm{ld}} \cup \cdots, \tag{21b}$$

$$\text{Running}: \quad \mathbb{R}^+ = \Omega_0^{\mathrm{f}} \cup I_0^{\mathrm{ld}} \cup \Omega_0^{\mathrm{sl}} \cup \Omega_1^{\mathrm{f}} \cup I_1^{\mathrm{rd}} \cup \Omega_1^{\mathrm{sr}} \cup \Omega_2^{\mathrm{f}} \cup I_1^{\mathrm{ld}} \cup \cdots. \tag{21c}$$

The conditions of activation of one mode have to be studied. For instance, conditions such that sticking occurs at an impact or during a step have been studied in Rubanovich and Formal'sky (1981), Hurmuzlu (1993), Chang and Hurmuzlu (1994) and Génot, Brogliato, and Hurmuzlu (1998). They evidently strongly depend on the process model like Amontons–Coulomb friction and the multiple impact restitution law. The concatenation of phases in (21) corresponds to desired invariant sets of the DES. From a general view point, any stability criterion should take into account both the non-smooth and the hybrid natures of such complementarity systems. As it is known, one may have several point of views of hybrid dynamical systems: continuous-time, discrete-event, or mixed, see e.g. Automatica (1999).

4.3. Constraints that guarantee foot sticking, stability margins

The control torques $u(\cdot)$ have to obey certain conditions to ensure sticking of the contacting feet at all times. These conditions can be written as (Génot, 1998; Génot et al., 1998)

$$A(q,\mu)u + B(q,\dot{q},\mu) > 0 \tag{22}$$

during smooth motion, and as

$$\bar{A}(q(t_k),\mu)\dot{q}(t_k^-) > 0 \tag{23}$$

at impact times (and for some choice of the double impact model), where μ is Amontons–Coulomb's friction coefficient at contact. The detailed calculations of inequalities (22) and (23) can be found in Génot, Brogliato, and Hurmuzlu (2001). Both inequalities in (22) and (23) are *necessary and sufficient* conditions to be satisfied by the control input u so that during the *whole motion* (*smooth and non-smooth*) sticking is maintained and detachment is monitored. The inequality in (22) can be rewritten as $A(\lambda) \geqslant 0$ and can be used to compute *stability margins* (Wieber, 2002). The notion of stability margin has been introduced correctly in Seo and Yoon (1995), who formulated a set of constraints in the spirit of (22), (23). The "distance" from the trajectory to the constraint boundary is defined as the maximal magnitude of a disturbance that is applied on the biped. Stability margins help understanding the difference between *static gait* (center of gravity located within its base of support), and *dynamic gait* (center of gravity may fall outside the support base) (Vaughan, 2003). To the best of our knowledge, none of the control laws proposed in the literature until now was shown to be "stable" with respect to these fundamental conditions, mainly due to the fact that the underlying dynamics of the robot does not capture the unilateral feature of the feet-ground contacts. To summarize, a controller which guarantees both (22) and (23) implies that the system evolves in the DES path in (21a).

4.4. Poincaré maps and stability

Generally, the approach to the stability analysis takes into account two facts about bipedal locomotion: the motion is discontinuous because of the impact of the limbs with the walking surface (Hurmuzlu & Moskowitz, 1987; Hurmuzlu, 1993; Katoh & Mori, 1994; Zheng, 1989; Grizzle et al., 2001), and the dynamics is highly nonlinear and non-smooth and linearization about vertical stance generally should be avoided (Vukobratovic et al., 1990; Hurmuzlu, 1993; Grizzle, Abba & Plestan, 1999).

A classical technique to analyze dynamical systems is that of Poincaré maps. In the three-link bipedal model of Section 2, we have shown that periodic motions of a simple biped can be represented as closed orbits in the phase space. Fig. 3(d) depicts a first return map obtained from the points of the trajectory that coincide with the instant of heel strike. A Poincaré map for a generalized coordinate ϕ_1 (which is typically a joint angle in bipedal systems) at the instant of heel strike now can be obtained by plotting the values of ϕ_1 at ith versus the values at $(i+1)$th heel strike.

One can choose an event such as the mere occurrence of heel strike, to define the Poincaré section (Hurmuzlu & Moskowitz, 1987; Kuo, 1999). We can construct several mappings depending on the type of motion. In general, however, the section can be written as follows:

$$\Sigma_i^+ = \{(q,\dot{q}) \in \mathbf{R}^{2p} \mid t = t_k^c\}, \quad i = 1,\ldots,l, \tag{24}$$

where the condition establishes the Poincaré section.

The discrete map obtained by following the procedure described above can be written in the following general form:

$$\xi_i = \mathbf{P}(\xi_{i-1}), \tag{25}$$

where ξ is a reduced dimension state vector, and the subscripts denote the ith and $(i-1)$th return values, respectively. Periodic motions of the biped correspond to the fixed points of $\mathbf{P}$ where

$$\xi^* = \mathbf{P}^k(\xi^*), \tag{26}$$

where $\mathbf{P}^k$ is the kth iterate. The stability of $\mathbf{P}^k$ reflects the stability of the corresponding flow. The fixed point ξ^* is said to be stable when the eigenvalues v_i, of the linearized map,

$$\delta\xi_i = \mathbf{DP}^k(\xi^*)\delta\xi_{i-1} \tag{27}$$

have moduli less than one. This technique employed in Hurmuzlu and Moskowitz (1987), Hurmuzlu (1993), François and Samson (1998), Kuo (1999), Grizzle et al. (1999, 2001), Piiroinen and Dankowicz (2002), Dankowicz and Piiroinen (2002), Dankowicz, Adolfsson and Nordmark (2001), Piiroinen, Dankowicz, and Nordmark (2001, 2003) and Quint van der Linde (1999) has several advantages. Using this approach the stability of gait conforms with the formal stability definition accepted in nonlinear mechanics. The eigenvalues of the linearized map (Floquet multipliers) provide quantitative measures of the stability of bipedal gait. Finally, to apply the analysis to locomotion one only requires the kinematic data that represent all the relevant degrees of freedom. No specific knowledge of the internal structure of the system is needed. This feature also makes it possible to extend the analysis to the study of human gait. Using this approach one can develop quantitative measures for clinical evaluation of the human gait (Hurmuzlu & Basdogan, 1994; Hurmuzlu, Basdogan, & Stoianovici, 1995).

5. Control of bipedal robots

The control problem of bipedal robots can be defined as choosing a proper input u in $(\mathscr{S})$ such that the system behaves in a desired fashion. The key issue of controlling the motion of bipeds still hinges on the specification of a desired motion. There are numerous ways that one can specify the desired behavior of a biped, which in itself is an open question. The control problem can become very simple or extremely complex depending on the specified desired behavior and the structure of the system. Typical bipedal machines are designed to perform tasks that are not confined to simple walking actions. Such tasks may include maneuvering in tight spaces, walking or jumping over obstacles, and running. In this article, we place our main focus on tasks that are primarily related to walking. Therefore, we will not be concerned with actions such as performing manipulation tasks with the upper limbs.

5.1. Passive walking

Possibly the first bipedal walking machine was built by Fallis (1888). The idea of a biped walking without joint actuation during certain phases of locomotion was initially proposed in Mochon and McMahon (1980a). The authors termed this type of walk as “ballistic walking”. The inspiration of this idea originated from evidence in human gait studies, which pointed out to relatively low levels of muscle activity in the swing limb during the swing period (see Basmajian, 1976; Zarrugh, 1976). Two planar models were used: (1) a three-element model with a single-link stance and two-link swing leg, (2) a four-element model with two-link stance and swing limbs. In each case, the authors searched for initial conditions at the onset of the swing phase such that the subsequent motion satisfied a set of kinetic and kinematic constraints. Then, they isolated the initial conditions that satisfied the imposed constraints. The simulation results were compared to experimentally measured knee angles and ground reaction forces. They concluded that their outcomes and human data had the same general shape. In Mochon and McMahon (1980b), they improved their model by adding the stance knee (two-element stance limb). With this more sophisticated model, the authors analyzed the model response by using three out of the five gait determinants (Saunders, Inman, & Eberhart, 1953). The premise of walking without joint actuation, prompted McGeer (1990) to propose the “passive walking”. McGeer developed numerical as well as experimental models of bipeds inspired by Fallis (1888) that have completely free joints. He demonstrated that these simple, unactuated bipeds can ambulate on downward planes only with the action of gravity.

Now, returning to (1)–(4), the typical passive control scheme is concerned with the free dynamics of the system $(\mathscr{S})$ given in Section 3.1 (i.e. the dynamics of the system subject to $u=0$). Then, as we have shown in Section 4.1, a Poincaré section Σ^+ can be selected (see (24)) to obtain a nonlinear mapping in the form of

$$\boldsymbol{\xi}_i = \mathbf{P}(\boldsymbol{\xi}_{i-1}, \varphi), \tag{28}$$

where the parameter vector φ typically includes the slope of the walking surface, member lengths, and member weights. Then, the underlying question becomes the existence and stability of the fixed points of this map (Kuo, 1999) and the resemblance of the resulting motions to bipedal walking. This task is generally very difficult to realize with the exception of very simple systems. For example, in the case of vibro-impact systems, analytical expressions to show existence can be found (Shaw & Shaw, 1989). In the case of slightly more complex systems, such explicit calculations become impossible since the free-motion dynamics is no longer integrable. Then one has to rely on numerical tools to derive both the Poincaré mapping and its local stability (Kuo, 1999; Piiroinen et al., 2001, 2003; Piiroinen & Dankowicz, 2002; Dankowicz & Piiroinen, 2002).

The main energy loss in these bipeds is due to the repetitive impacts of the feet with the ground surface. The gravitational potential energy provides the compensation for this loss, thus resulting in steady and stable locomotion for certain slopes and configurations. The ground impacts (Hurmuzlu & Moskowitz, 1986) provide a unique mechanism that leads to stable progression in very simple bipeds as has been recently demonstrated by several investigators. In Goswami, Thuilot, and Espiau (1996, 1998), the authors consider a simple model that includes two variable length members with lumped masses representing the upper body and the two limbs. A third lumped mass is attached to this point, which represents the upper body. They analyze the nonlinear dynamics of (28) subject to prescribed variations in the elements of φ. They primarily focus on the effect of the ground slope, mass distribution, and limb lengths. Numerical analysis of the nonlinear map, results in the detection of stable limit cycles as well as chaotic trajectories that are reached through period doubling cascade. A simpler, two-link model was considered in Garcia et al. (1997). They also demonstrated that this simple biped can produce stable locomotion as well as very complex chaotic motions reached through frequency doubling cascade. Chatterjee and Garcia (1998) and Das and Chatterjee (2002) studied the existence of periodic gaits in the limit of zero slope. The addition of passive arms served to reduce side-to-side rocking in a 3D passive walker (Collins, Wisse, & Ruina, 2001). Quint van der Linde (1999) includes phasic muscle contraction as the energy source, and vary the muscle model parameters to create new periodic gaits. The main challenge of the study of passive gait is to translate the understanding gained by studying passive systems to active systems. The MIT Leg Lab planar bipeds are controlled this way (Pratt, 2000) for periodic walking. In other words, the desired trajectories $q_d(t)$ are designed from the study of passive walking. This is not the case for the Honda robots where $q_d(t)$ are obtained from human recordings. One of the main obstacles to build bipedal robots remains to be the prohibitively high joint torques that are often required to realize even routine walking tasks. A comprehensive investigation that bridges the passive studies to better design of active control schemes would be a natural extension of passive locomotion research. This is achieved in Piiroinen and Dankowicz (2002), Dankowicz and Piiroinen (2002), who propose a control method based on discrete adjustments of the swing-foot orientation prior to contact, hence indirectly affecting the nature and timing of the subsequent impact. This results in the (local) stabilization of a motion that is naturally occurring in the system.

5.2. Walking with active control

5.2.1. Controller design

The control action must assure that the motion of a multi-link kinematic chain, which can characterize a typical biped, is that of a walker. Although, the characteristics of the motion of a walker is still open to interpretation, we may translate this requirement to a set of target/objective functions given in the form

$$g_i(q(t), \dot{q}(t), q_d(t), \dot{q}_d(t), \lambda_n, \lambda_t, \Theta, u) = 0 \quad i = 1, \ldots, k \leqslant p, \tag{29}$$

where Θ is a vector of parameters that prescribes certain aspects of the walking action such as progression speed, step length, etc. Note that for the sake of simplicity of the notations q will denote in the sequel the vector of generalized coordinates of the model considered by the referenced authors, i.e. either of the full order model, or of the reduced order model when assuming that the ground contacts, when active, are bilateral contacts.

The control problem can be described as specifying the vector of joint actuator torques u in (1) such that the system behaves in a certain way. The simplest way to proceed is to specify the time profiles of the joint trajectories. Investigators in the field used kinematics of human gait as desired profiles (see Hemami & Farnsworth, 1977; Khosravi, Yurkovich, & Hemami, 1987; Vukobratovic et al., 1990). One can also simply specify certain aspects of locomotion such as walking speed, step length, upright torso, etc. (Chudinov, 1980, 1984; Lavrovskii, 1979, 1980; Beletskii, 1975; Beletskii & Kirsanova, 1976; Beletskii & Chudinov, 1977b, 1980; Beletskii, Berbyuk, & Samsonov, 1982; Grishin & Formal'sky, 1990; Novozhilov, 1984; Hurmuzlu, 1993; Chang & Hurmuzlu, 1994; Yang, 1994). In Beletskii and Chudinov (1980), the authors use the components of the ground reaction forces in addition to kinematics in order to completely specify the control torques. Once the objective functions are specified, one has to choose a control scheme in order to specify the joint moments (control torques) that drive the system toward the desired behavior. We encounter several approaches to this problem that can be enumerated as follows:

(1) *Linear control*: The equations of motion are linearized about the vertical stance, assuming that the posture of the biped does not excessively deviate from this position. For example, in Jalics, Hemami, and Clymer (1997), Kajita and Tani (1996), Kajita, Yamaura, and Kobayashi (1992), Grishin, Formal'sky, Lensky and Zhitomirsky (1994), Zheng and Hemami (1984), Hemami, Zheng, and Hines (1982), Golliday and Hemami (1977), Hemami and Farnsworth (1977), Gubina, Hemami, and McGhee (1974), Mitobe, Capi, and Nasu (2000), Seo and Yoon (1995) and Garcia, Estremera, and Gonzales de Santos (2002), a PD controller was used to track joint trajectories. The linear controller, however, cannot track time functions. Thus, the authors discretized the desired joint profiles and let the controller track the trajectory in a point-to-point fashion. van der Soest "Knoek", Heanen, and Rozendaal (2003) study the influence of delays in the feedback loop on stance stability, including muscle model.

(2) *Computed torque control*: This method was applied (Hemami & Katbab, 1982; Lee & Liao, 1988; Hurmuzlu, 1993; Yang, 1994; Jalics et al., 1997; Lum et al., 1999; Song, Low, & Guo, 1999; Park, 2001; Taga, 1995: Chevallereau, 2003) to bipedal locomotion models with various levels of complexities. In Chevallereau (2003), the computed torque is combined with a time-scaling of the desired trajectories optimally designed (Chevallereau & Aoustin, 2001), which allows the finite-time convergence of the system's state towards the desired motion. The finite-time convergence especially allows one to avoid the tricky problems due to tracking errors induced by impacts (Bourgeot & Brogliato, 2003; Brogliato et al., 1997, 2000).

(3) *Variable structure control*: This method results in a feedback law that ensures tracking despite uncertainties in system parameters. In this approach, one chooses the control vector as

$$u_i = \hat{u}_i - k_i \operatorname{sign}(s), \tag{30}$$

where $\hat{u}_i$ is a trajectory tracking controller with fixed estimated parameters. The second term, is the variable structure part of the control input. The function s defines the sliding surface that represents the desired motion. This is a high gain approach that is advantageous because it ensures convergence in finite time. In locomotion, the stability of the overall motion relies on the effectiveness of the controller in eliminating the errors induced by impact during the subsequent step. The reader can check Chang and Hurmuzlu (1994) and Lum et al. (1999) to see the application of such a controller to a five-element planar model.

(4) *Optimal control*: Optimal control methods have been used by researchers to regulate the smooth dynamic phase of bipedal locomotion systems. Two approaches have been taken to the optimization problem. The first method is based on computing the values of selected parameters in the objective functions that minimize energy-based cost functions (Frank, 1970; Vukobratovic, 1976; Beletskii & Chudinov, 1977a; Beletskii et al., 1982; Rutkovskii, 1985; Channon, Hopkins, & Pham, 1992; Saidouni & Bessonnet, 2003). We note that optimal trajectory planning including the dynamics in (1)–(4) is equivalent to searching for an optimal open-loop control $u(t)$. The second approach is based on variational methods to obtain controllers that minimize cost functions (Beletskii & Bolotin, 1983; Bolotin, 1984; Furusho & Sano, 1990; Channon, Hopkins, & Pham, 1996a, b). It is the direct application of classical optimal control methods to bipedal locomotion, see Channon, Hopkins, & Pham (1996c) for the most advanced work in this topic. Here the authors regulate the motion of the biped over a support phase with a quadratic cost function. van der Kooij, Jacobs, Koopman, & van der Halm (2003) propose a model predictive controller designed from a tangent linearization to regulate gait descriptors formulated as end-point conditions. The main obstacle towards real implementation is a too large computation time.

(5) *Adaptive control*: The adaptive control approach has received very little attention in biped control. Perhaps it is does not have real advantage in controlling bipedal locomotion. Nevertheless, Yang (1994) has applied adaptive control approach to a three link, planar robot. Experiments have been led at the MIT Leg Lab (Pratt, 2000) using adaptive control.

(6) *Shaping discrete event dynamics*: The abrupt nature of impact makes it practically impossible to directly control its effect on the system state. Even an approximation of an impulsive Dirac input would demand actuators with too high bandwidth (to say nothing of induced vibrations in the mechanical structure). An alternate approach can be found in shaping the system state prior to the impact instant such that a desired outcome is assured. Such an approach was taken in Hurmuzlu (1993) and Chang and Hurmuzlu (1994). In these studies, a set of objective functions in the form of (29) was tailored. Assuming perfect tracking, the authors derived the expression for the system state immediately before the instant of impact in terms of the parameter vector Θ. Subsequently, the post-impact state was computed for specific values of the parameter vector. The parameter space was partitioned into regions according to slippage and contact conditions that result from the foot impact. Then, this partitioning was used to specify controller parameters such that the resulting gait pattern has only single support phase and the feet would not slip as a result of the feet impact. Dunn and Howe (1994) developed conditions in terms of motion and structural parameters such that they minimize/eliminate the velocity jumps due to ground impact and limb switching. Thus, in their case, the objective of the shaping was to remove the effect of the impact altogether. Miura and Shimoyama (1984) used a feedforward input that modifies the motion at the end of each step from measurements informations. Grizzle et al. (1999, 2001) and Werstervelt et al. (2003) have also used a similar approach. They shape the state before the impact, so that at the next step the state resides in the zero dynamics. Doing so they create a periodic gait that corresponds to the zero dynamics defined from a set of output functions. Piiroinen and Dankowicz (2002) locally stabilize a passive walk with a specific strategy, see Section 5.1.

(7) *Stability and periodic motions*: Stability of the overall gait is often overlooked in locomotion studies. Typically, controllers have been developed, and few gait cycles have been shown to demonstrate that the biped "walks" with the given controller. A thorough analysis of the nonlinear dynamics of a planar, five-element biped (Hurmuzlu, 1993) reveals a rich set of stable, periodic motions that do not necessarily conform to the classical period one locomotion. Tracking errors in the control action may lead to stable gait patterns that are different than the ones that are intended by the objective functions. One way to overcome this difficulty is to partition the parameter space such that one would choose specific values that lead to a desired gait pattern. This approach is taken in Hurmuzlu (1993) and Chang and Hurmuzlu (1994).

(8) *Other specialized control schemes*: Several investigators (Grishin & Formal'sky, 1990; Grishin et al., 1994; Beletskii, 1975; Chudinov, 1980, 1984; Katoh & Mori, 1994; Lavrovskii, 1979, 1980). used simplified models without impact and constructed periodic trajectories by concatenation of orbits obtained from individually controlled segments of the gait cycle. This approach is quite similar in spirit to the Kobrinskii (1965) method that is used the existence of trajectories of the impact damper and the impacting inverted pendulum (see Fig. 1). Blajer and Schielen (1992) compute a nonlinear feedforward torque corresponding to a "non-impacting" reference walk and use PD motion and PI force feedback to stabilize around the reference trajectory. Fuzzy logic control was used (Shih, Gruver, & Zhu, 1991) to develop a force controller that regulates ground reaction forces in swaying actions of an experimental biped. A group of investigators changed the parameters in the objective functions such that the desired motion is adapted to changing terrain conditions (Igarashi & Nogai, 1992; Shih & Klein, 1993; Zheng & Sheng, 1990). Zheng (1989) used an acceleration compensation method to eliminate external disturbances from the motion of an experimental eight joint robot. Kuo (1999) derives numerically an impact Poincaré map that represents the walking cycle, and proposes a linear state feedback that stabilizes this cycle. Clearly, this is conceptually completely different from the works described above (see item (1) *Linear control*) since the design is based on a linearization of the Poincaré map itself and not of the continuous dynamics on one step.

5.2.2. DES stabilization

As we have shown in Section 4.2, walking corresponds to a particular sequence of activations of the modes of the DES associated to the biped seen as a complementarity mechanical system. Such a sequence can be seen as an invariant set of the DES dynamics, see (21a). These control techniques aim at stabilizing this invariant set in the sense that the robot should ultimately be able to recover from falls and restart walking (Fujiwara et al., 2002). Notice that this approach does not emphasize the low-level details of the walk (walking speed, steps length, etc.). An interesting approach in the area is the zero moment point (ZMP) method proposed first by Vukobratovic and his co-workers (Vukobratovic & Juricic, 1969; Vukobratovic et al., 1990). The reader can refer to Goswami (1999) and Wieber (2002) and references within for a detailed discussion regarding the real meaning of ZMP, and to Garcia et al. (2002) for a complete description of equivalent stability concepts. Several different, but equivalent, definitions of the ZMP are given (Hemami & Farnsworth, 1977; Takanishi, Ishida, Yamaziki, & Kato, 1985; Arakawa & Fukuda, 1997; Hirai, Hirose, & Kenada, 1998). The simplest one is (Hemami & Farnsworth, 1977) *the point where the vertical reaction force intersects the ground*, i.e. the center of pressure. The ZMP stability criterion states that the biped will not fall down as long as the ZMP remains inside the convex hull of the foot-support. In these studies, the authors impose the motion of the lower limb kinematics from human kinematic data, which they term synergies. This way, the ZMP criterion is used to switch between low-level controllers (which satisfy some objective functions like trajectory tracking), so as to stabilize the DES orbit in (21a) (Park, 2001), and possibly avoid obstacles (Yagi & Lumelsky, 2000). The ZMP method was also applied with other controllers that are not based on prescribing human data (Borovac, Vukobratovic, & Surla, 1989; Fukuda, Komota, & Arakawa, 1997; Shih, Gruver, & Lee, 1993; Mitobe et al., 2000; Park, 2001; Huang et al., 2001; Vanel & Gorce, 1997). One of the best example of the high degree of efficiency that such control approaches are able to attain are the bipeds constructed by Honda (Hirai et al., 1998; Hirai, 1997; Pratt, 2000), whose control mainly rely on a suitable combination of local linear controller with high-level (or logical) conditions. In Pratt, Chew, Torres, Dilworth, and Pratt (2001), an intuitive approach for making some bipedal machines walk is proposed. It is based on the so-called virtual model control. The need for both low- and high-level control together with on-line desired trajectories planning is explained in El Hafi and Gorce (1999) and Vanel and Gorce (1997) where only the supervisory aspects are studied. Wieber (2002) proposes a quite interesting study, starting from (22). A general criterion for the DES path (21a) stability (equivalently, its invariance) is established, and the link with Lyapunov functions is made (excepting impacts). Stability margin (roughly, the distance from the actual trajectory to the boundary of an admissible set of trajectories, outside of which the robot falls down (Seo & Yoon, 1995) can be derived. Such studies are of primary importance for characterizing the stability of rehabilitated paraplegics (Popovic et al., 2000).

6. Conclusions and directions for future research

This survey is devoted to the problem of modeling and control of a class of non-smooth nonlinear mechanical systems, namely bipedal robots. It is proposed to recast these dynamical systems in the framework of mechanical systems subject to complementarity conditions. Unilateral constraints that represent possible detachment of the feet from the ground and Coulomb friction model can be written this way. In the language of Full and Koditschek (1999), this is a suitable *template*. Such a point of view possesses several advantages:

(1) It provides a unified approach for mathematical, numerical and control investigations. This is a quite important point since numerical studies are mandatory in any mechanical and/or control design.
(2) This framework encompasses all the models which have been used to study locomotion in the control and robotics literature.
(3) Though we restrict ourselves to rigid body contact/impact models, lumped flexibilities can easily be introduced, both at the contact or in the structure itself (flexible joints). Introducing flexibilities may be necessary (Pratt, 2000; Pratt & Williamson, 1995), and is physiologically sound (Gunther & Blickman, 2002). This will, however, make the control problem harder to solve and may be ineffective in walking (Kar et al., 2003). It may also create serious difficulties in the analysis, especially with multiple contacts (Paoli & Schatzman, 2002).
(4) Such models have proved to predict quite well the motion as several experimental validations available in the literature show (Abadie, 2000).
(5) As shown in this survey, the proposed modeling approach allows one to clarify which stability tools one may use to characterize the stability of a bipedal robot.
(6) Finally, it is the opinion of the authors that one important development is still missing in the field of biped design: a concise and sufficiently general theoretical analysis framework, based on realistic models, that allows the designer to derive stable controllers taking into account the hybrid dynamics in their entirety.

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