June 11, 2008

# Erratum to *Numerical Methods for Nonsmooth Dynamical Systems: Applications in Mechanics and Electronics*

- Figure 1.19: the caption inside the four figures is "Implicit Euler", not "Explicit Euler".
- Page 127, equation after (3.84), in the brackets: this is $\frac{1}{m_0}$, not $\frac{1}{m_S}$.
- Section 7.1.2.1, equation (7.10): The correct numbering of the functions $h_i(\cdot)$ is reversed, i.e. $h_1(x) = -c_1(x) + c_2(x)$, $h_2(x) = c_1(x) + c_2(x)$, $h_3(x) = c_1(x) - c_2(x)$, $h_4(x) = -c_1(x) - c_2(x)$.
- **About Stewart's event-driven method in section 7.1:** Stewart has published another paper:
  - D.E. Stewart, "A numerical method for frcition problems with multiple contacts", J. Australian Math. Soc. Ser. B, vol.37, pp.288-308, 1996.

  in which he shows that the framework developed in his 1990 paper has some limitations. For instance a simple system like $\dot{x}_1(t) \in -\text{sgn}(x_1(t))$, $\dot{x}_2(t) \in -\text{sgn}(x_2(t))$, that looks quite simple and actually is, fails to satisfy the assumptions on which Theorem 7.9 relies. Stewart proposes a modification of the way the sets $R_i$ in (7.8) are defined.
- Page 243, line 3: $f(x,t)$.
- Section 9.3.3, let us show how one can prove the uniqueness of solutions of the one-step nonsmooth problem in (9.56) without going through the complementarity formalism. For that we first write compactly the sign inclusion in (9.54) as $u \in \text{Sgn}(-y)$, that is $u \in -\text{Sgn}(y)$, where $u$ and $y$ are $m-$dimensional vectors. Obviously the $\text{Sgn}(\cdot)$ function is monotone. The discretized relay system then becomes:

$$\begin{cases} x_{k+1} = x_k + hAx_{k+1} + hBu_{k+1} \\ 0 \in u_{k+1} + \text{Sgn}(Cx_{k+1} + Du_{k+1}) \end{cases} \tag{1}$$

Let us first study the case $D = 0$. In this case one can easily rewrite (1) as

$$0 \in -x_k + (I_n - hA)x_{k+1} + hB\text{Sgn}(Cx_{k+1}) \tag{2}$$

Let us assume that there exists $P \in \mathbb{R}^{n\times n}$, $\text{rank}(P) = n$, such that $PB = C^T$. Then we can rewrite equivalently the inclusion (2) as

$$0 \in -P^{-1}x_k + P^{-1}(I_n - hA)x_{k+1} + hP^{-1}PB\text{Sgn}(Cx_{k+1}) \quad \Leftrightarrow \quad 0 \in -P^{-1}x_k + P^{-1}(I_n - hA)x_{k+1} + hP^{-1}C^T\text{Sgn}(Cx_{k+1}) \tag{3}$$

Simplifying the $P^{-1}$ term we obtain

$$0 \in -\frac{1}{h}x_k + \frac{1}{h}(I_n - hA)x_{k+1} + C^T \mathrm{Sgn}(Cx_{k+1}) \quad (4)$$

For small enough $h > 0$ the matrix $(I_n - hA)$ is positive definite. The multifunction $x \mapsto C^T \mathrm{Sgn}(Cx)$ from $\mathbb{R}^n$ into $\mathbb{R}^n$ is monotone. Since addition preserves monotonicity, we conclude that the right-hand-side of (4) is monotone, it is even strongly monotone since $I_n - hA > 0$. Thus the inclusion (or generalized equation) (4) has a unique solution by Theorem ?? in Facchinei and Pang (2003).

Suppose now that $D > 0$ (and thus has full rank $m$). Let us do the variable change $w = Cx_{k+1} + Du_{k+1}$ so that $u_{k+1} = D^{-1}(w - Cx_{k+1})$. Thus we get

$$0 \in D^{-1}(w - Cx_{k+1}) + \mathrm{Sgn}(w) \quad (5)$$

which is an inclusion with unknown $w$. Since $D^{-1}$ is positive definite, we conclude similalry that the right-hand-side of (5) is strongly monotone and $w$, thus $u_{k+1}$, is unique. So $x_{k+1}$ is unqiue as well.

Let us now investigate the general case. Easy manipulations of (1) result in the inclusion

$$0 \in u_{k+1} + \mathrm{Sgn}\left[(hC(I_n - hA)^{-1}B + D)u_{k+1} + C(I_n - hA)^{-1}x_k\right] \quad (6)$$

Let us denote $H(h) = hC(I_n - hA)^{-1}B + D \in \mathbb{R}^{m \times m}$. Suppose that $H(h)$ is full-rank $m$ for some small enough $h > 0$. Notice that when $D = 0$ this is equivalent to have $I_n - hA$ full-rank $n$, and $C \in \mathbb{R}^{m \times n}$ full column rank $m$, and $B \in \mathbb{R}^{n \times m}$ full row rank $m$. When $D$ is full rank $m$ it suffices that $h$ be small enough. We can rewrite equivalently (6) as

$$0 \in H^T(h)u_{k+1} + H^T(h)\mathrm{Sgn}\left[H(h)u_{k+1} + C(I_n - hA)^{-1}x_k\right] \quad (7)$$

To guarantee the uniqueness of solutions to this inclusion we may once again rely on monotonicity arguments. It is then sufficient that $H(h) > 0$ to get the strong monotonicity of the right-hand-side. In fact the P-matrix property suffices. We retrieve the condition of Theorem 9.22. We see that the treatment of the general case results in more stringent assumptions on the data. In the case $D = 0$ the only assumption on which uniqueness relies is $PB = C^T$ for some full rank matrix $P$, and this doesn't mean that $H(h) = hC(I_n - hA)^{-1}B = hB^T P(I_n - hA)^{-1}B$ possesses any particular property other than full rank $m$ if $B$ has full column rank $m$ and $h$ is small enough.

- Page 270, line 2 of equation (9.59), this is $(I_n - hA)^{-1}hB$.
- In the bibliography:
  - M. Anitescu, G. Lesaja, F. Potra. Equivalence between different formulations of the linear complementarity problem. Technical Report 71, University of Iowa Technical Reports in Computational Mathematics, Iowa City, IA 52242, USA, 1995.

    has appeared as
  - M. Anitescu, G. Lesaja, F. Potra. Equivalence between different formulations of the linear complementarity problems, Optimization Methods and Software (OMS), 7 (3-4), pp. 265-290, 1997. DOI: 10.1080/10556789708805657
- **About the numerical simulation of nonsmooth electrical circuits:** a quite interesting paper that is unfortunately not included in the bibliography is
  - F. Yuan, A. Opal, "Computer methods for switched circuits", IEEE Transactions on Circuits and Systems – I: Fundamental Theory and Applications, vol.50, no 8, pp.1013-1024, August 2003.

An excerpt from this survey paper:

*In time-domain analysis, we have shown that SPICE-like modeling of switches leads to stiff systems that are both ill-conditionned and expensive to simulate. Ideal modeling of switches greatly simplifies circuit configuration and speeds up simulation. It, however, gives rise to inconsistent initial conditions that are difficult to be handled by conventional integration methods.*

The complementarity methods described in the book allows one to solve the inconsistent initial conditions issue.