Technical Notes and Correspondence

Adaptive Hybrid Force-Position Control for Redundant Manipulators

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Abstract—**This note presents an adaptive control scheme for manipulators with redundant degrees of freedom. The control purpose is to achieve a desired interaction force between the end-effector and the environment as well as to regulate the robot tip position in the Cartesian space. This control approach does not require measurement of the joint acceleration nor the force derivative.**

I. Introduction

The last ten years have witnessed an increasing interest in adaptive control of robot manipulators. In particular, adaptive motion control of rigid robots has attained a certain degree of maturity [1]–[9]. These studies have been carried out assuming that the robot moves freely in the task space. Nevertheless, the interaction between the robot and the environment has to be taken into account in most of the real industrial applications of articulated mechanical systems. This naturally leads us to consider control schemes that regulate the interaction forces between the end-effector and the environment. Several types of such controllers have been proposed in the literature and can basically be classified as complaint motion control [10]–[12], pure force control [3], [17], and hybrid schemes [3], [10], [12]–[15], and [18].

Recently, the first adaptive force controllers have been proposed in [3] and [17]. One of the problems encountered in [3] was the use of the first derivative of the measured force in the control law. Such problem has been solved in [17] for the case of single-link mechanical manipulators. To date, the extension of such an approach to more general manipulators does not seem feasible in view of the robot redundancies.

In this note, we present an adaptive hybrid force-position control scheme applicable to general redundant mechanical manipulators. As is currently assumed, the robot is supposed to work in a space free of singularities, i.e., a space where the Jacobian matrix is full rank. The control input does not require measurement of the force first derivative nor the joint acceleration. The stability analysis is based on the robot's passivity properties.

This note is organized as follows. Section II presents the robot dynamic model and problem formulation. Section III is devoted to develop the control scheme for the known-parameters case. The adapative controller study is given in Section IV. The concluding remarks are finally given in Section V.

II. Dynamics of Rigid Robots

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Based on Euler–Lagrange equations [10], the mechanical manipulators dynamic model is given by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau - \tau_e \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \mathscr{R}^n$ are the joint position, velocity, and acceleration, $D(q)(n \times n)$ is the inertia matrix, $C(q,\dot{q})(n \times n)$ contains the Coriolis and centrifugal terms, $g(q)$ is the n-dimensional gravitational torque vector, $\tau(n \times 1)$ is the actuators torque, and τ_e is the interaction torque with the environment.

The dynamic model (1) has the following fundamental properties [7].

P1) The inertia matrix $D(q)$ is symmetric, positive definite, and both $D(q)$ and $D^{-1}(q)$ are uniformly bounded as a function of q.

P2) All the constant inertial parameters in $D(q), C(q,\dot{q})$, and $g(q)$ appear as coefficients of known functions of the generalized coordinates [24], [25].

P3) $N(q,\dot{q}) = (\dot{D}(q) - 2C(q,\dot{q}))$ is a skew symmetric matrix (i.e., $N = -N^T$).

The end-effector position and orientation in the Cartesian space, $x \in \mathscr{R}^m$ and its derivative $\dot{x} \in \mathscr{R}^m (m \le 6)$ are related to the joint coordinates q and velocities $\dot{q}$ by

$$x = f(q) \tag{2}$$

$$\dot{x} = J(q)\dot{q} \tag{3}$$

where $J(q)$ is the $m \times n$ $(m \le n)$ Jacobian matrix of the robot direct kinematic relation $f(q)$.

The interaction force $\bar{F} \in \mathscr{R}^m$ at the end-effector, performing work on x, and the interaction torque $\tau_e \in \mathscr{R}^n$ in the joint space are related by [19]

$$\tau_e = J^T(q)\bar{F}. \tag{4}$$

Remark 1: If the end-effector orientation is defined using the Euler angles, then the vector of linear and angular velocities $w = [v, \Omega]$ is related to $\dot{x}$ by $w = T(x)\dot{x}$ (see [15] and [18]) and $w = \bar{J}(q)\dot{q}$ where $\bar{J}$ is the Jacobian matrix. In this case, (3) will still hold with $J = T^{-1}\bar{J}$. We will assume that $\bar{F}$ is measurable. However, if instead we measure $\bar{F}_c$, performing work on the axis of the Cartesian frame, then the transformation matrix T can be used to compute $\bar{F}$. ▽

Let us define x_t as the position and orientation vector in the task frame in the same way as x. Then the following relation holds [18]:

$$\dot{x}_t = R(x, x_t)\dot{x} \tag{5}$$

where $R(x, x_t)$ is a transformation matrix–assumed to be nonsingular—which depends on the environment geometry. This transformation is introduced to simplify the relationship between the interaction force and the environment deformation defined below.

Throughout the note, we will assume that the interaction force $\bar{F}_t$ in the task space is proportional to the environment deformation $x_t - x_{e_t}$, i.e.,

$$\bar{F}_t = \bar{K}(x_t - x_{e_t}) \tag{6}$$

with

$$\bar{F} = R^T(x, x_t)\bar{F}_t \tag{7}$$

where $\bar{K}$ is the $(m \times m)$ constant stiffness matrix and x_{e_t} is the environment/end-effector contact point position when the robot does not exert any force on the environment. This assumption clearly requires the environment to be homogeneous (i.e., composed of materials having the same elasticity properties) and also requires the shape of the environment to be such that the stiffness does not depend on the contact point. This assumption will be required in the design of an adaptive control scheme in the next section. It is clear that if the effect of the environment's shape on the stiffness matrix is known, it can be incorporated in expression (6) as long as the unknown parameters appear linearly.

Here, for simplicity x_{e_t} is considered constant. In the case when x_{e_t} is time-varying the analysis is still valid if the relationship between x_{e_t} and x is known.

In general, the stiffness matrix $\bar{K}$ in (4) is not necessarily invertible. We will assume without loss of generality, that the elements of $\bar{F}_t$ have been chosen in such a way that $\bar{F}$ can be decomposed as

$$\bar{F} = \left[\frac{F_t}{F_t'}\right] = \left[\frac{K}{K'}\right](x_t - x_{e_t}) \tag{8}$$

where $F_t \in \mathscr{R}^s$, K is a full-rank $(s \times m)$ $(s \leq m)$ matrix and the rows of K' depend linearly on the rows of K. Therefore, we will only be able to control F_t, the elements F_t' being given as linear combinations of those of F_t.

In the next section, we will present a force position control scheme for robot manipulators. The control scheme requires only measurement of $q, \dot{q}$, and $\bar{F}$. Section III gives the control scheme structure for the case when the system parameters are known. Section IV presents the corresponding adaptive scheme and studies its convergence properties.

III. Force / Position Control Scheme: The Known Parameters Case

We will consider that the following assumptions are satisfied:

A1) The Jacobian $J(q)$ in (3) is a full-rank known matrix.

A2) The transformation matrix $R(x)$ in (5) is known.

A3) The stiffness matrix K in (8) is a constant matrix.

A4) $q, \dot{q}$, and $\bar{F}$ are physically measurable.

In this section, we will present a nonadaptive control scheme for the case when all the system parameters are known, i.e., $D(q), C(q, \dot{q})$ are known.

The following identities will be used in the control law synthesis:

$$I = J^+J + J- \tag{9}$$

$$I = K^+K + K- \tag{10}$$

where

$$J^+ = J^T(q)[J(q)J^T(q)]^{-1} \tag{11}$$

$$K^+ = K^T[KK^T]^{-1} \tag{12}$$

$$J^- = I - J^T(q)[J(q)J^T(q)]^{-1}J(q) \tag{13}$$

$$K^- = I - K^T[KK^T]^{-1}K. \tag{14}$$

J^+ and K^+ are the Penrose pseudo inverse and J^- and K^- are projectors, i.e., idempotent matrices

$$J^-J^- = J^- \tag{15}$$

$$K^-K^- = K^-. \tag{16}$$

Note also that

$$JJ^+ = I \tag{17}$$

$$JJ^- = 0 \tag{18}$$

$$KK^+ = I \tag{19}$$

$$KK^- = 0. \tag{20}$$

Identity (9) allows us to decompose $\dot{q}$ in (3) as follows:

$$\dot{q} = J^+J\dot{q} + J^-\dot{q} = J^+\dot{x} + J^-\dot{q} \tag{21}$$

where $J^-\dot{q}$ is in the null space of J, [see (18)] and $J^+\dot{x}$ is in its complement [see (17)]. Similarly, from (5), (8), and (10)

$$\dot{x} = R^{-1}\dot{x}_t \tag{22}$$

$$\dot{x}_t = K^+K\dot{x}_t + K^-\dot{x}_t \tag{23}$$

$$\dot{x}_t = K^+\dot{F}_t + K^-\dot{x}_t \tag{24}$$

$$\dot{x} = R^{-1}\left(K^+\dot{F}_t + K^-\dot{x}_t\right). \tag{25}$$

The above procedure can also be used to decompose $\ddot{q}$. Taking the first derivative of (3)

$$\ddot{x} = J\ddot{q} + \dot{J}\dot{q}. \tag{26}$$

Using identity (9) and the above

$$\ddot{q} = J^+J\ddot{q} + J^-\ddot{q} = J^+[\ddot{x} - \dot{J}\dot{q}] + J^-\ddot{q}. \tag{27}$$

Similarly from (8) and (10)

$$\ddot{x}_t = K^+K\ddot{x}_t + K^-\ddot{x}_t = K^+\ddot{F}_t + K^-\ddot{x}_t. \tag{28}$$

From (5) we have

$$\ddot{x} = R^{-1}\left(\ddot{x}_t - \dot{R}\dot{x}\right). \tag{29}$$

Introducing (2), (3), (21), and (25) through (29) into (1) we obtain

$$D\left\{J^+\left[R^{-1}\left(K^+\ddot{F}_t + K^-\ddot{x}_t - \dot{R}J\dot{q}\right) - \dot{J}\dot{q}\right] + J^-\ddot{q}\right\}$$
$$+ C\left\{J^+R^{-1}\left(K^+\dot{F}_t + K^-\dot{x}_t\right) + J^-\dot{q}\right\} + g = \tau - \tau_e. \tag{30}$$

This equation is exactly equivalent to (1), however, (30) enables to clearly distinguish those components of F_t, x_t, and q that can effectively be controlled. The following lemma proposes a control law that regulates the force F_t and the component of the task-space position x_t in the null space of K (i.e., K^-x_t).

Lemma 1: Consider the mechanical manipulator model in (30). Assume that the control input τ is given by

$$\tau = \tau_{\text{ideal}}$$
$$= \tau_e + D\left\{J^+R^{-1}\left[K^+\dot{v}_f + K^-\dot{v}_x - \dot{R}J\dot{q} - R\dot{J}\dot{q}\right] + J^-\ddot{q}_d\right\}$$
$$+ C\left\{J^+R^{-1}[K^+v_f + K^-v_x] + J^-\dot{q}_d\right\} + g - Dz - Gr \tag{31}$$

where

$$z = \frac{d(J^+R^{-1})}{dt}\left\{K^+\left(\dot{F}_t - v_f\right) + K^-(\dot{x}_t - v_x)\right\} + \frac{dJ^-}{dt}\dot{\tilde{q}} \quad (32)$$

$$r = J^+R^{-1}\left[K^+\left(\dot{F}_t - v_f\right) + K^-(\dot{x}_t - v_x)\right] + J^-\dot{\tilde{q}} \quad (33)$$

$$v_f = \dot{F}_d - \lambda\tilde{F}; \qquad \lambda > 0 \quad (34)$$

$$v_x = \dot{x}_d - \lambda\tilde{x}; \qquad \lambda > 0 \quad (35)$$

$$\tilde{F} = F_t - F_d \quad (36)$$

$$\tilde{x} = x_t - x_d \quad (37)$$

$$\tilde{q} = q - q_d \quad (38)$$

where q_d, x_d, F_d, and their first and second derivatives are the desired bounded values for q, x_t, F_t, and their respective derivatives satisfying

$$F_d^{(i)} = K(x_d^{(i)} - x_{ed}^{(i)}) \qquad \text{for } i = 0, 1, 2. \quad (39)$$

$G = G^T > 0$ is a constant matrix and λ is a positive constant.

Introducing the control input in (31) through (37) into the system (30) gives the following closed-loop error equation

$$D\dot{r} + Cr + Gr = 0. \quad (40)$$

Therefore, $r \in L_2 \cap L_\infty, \dot{q} \in L_\infty$ and $\dot{r} \in L_\infty$ which implies that $r \to 0$ and finally, $\tilde{F}$, $K^-\tilde{x}$, and $J^-\tilde{q}$ converge to zero as $t \to \infty$ and $\dot{\tilde{F}}$ and $K^-\dot{\tilde{x}}$ are in L_2. ▽

The proof is given in Appendix A.

Remark 2: Lemma 3 uses partitions of both the environment's stiffness and the manipulator's Jacobian to achieve a desired contact interaction force. In view of redundancies, there are some components of x and $\dot{q}$ denoted K^-x and $J^-\dot{q}$, respectively, which can be controlled independently of the interaction force. This fact is exploited in the controller design presented in Lemma 3. The next lemma states that the ideal control input proposed in Lemma 1 can in fact be given as an expression that is linear in the parameters. This property is essential to the development of an adaptive control scheme in the next section.

Lemma 2: The ideal control input in Lemma 1, (31) can be rewritten as

$$\tau_{\text{ideal}} = \tau_e + Y(r, \overline{F}, q, \dot{q})\theta \quad (41)$$

where $\theta \in \mathscr{R}^p$ is a constant parameter vector and Y is a known $(n \times p)$ matrix function of K, $\overline{F}$, q, $\dot{q}$, q_d, $\dot{q}_d$, $\ddot{q}_d$, x_d, $\dot{x}_d$, $\ddot{x}_d$, F_d, $\dot{F}_d$, $\ddot{F}_d$, and r. ▽

The proof is outlined in Appendix D.

IV. Adaptive Force / Position Control Scheme

In this section, we present an adaptive force/position control law for mechanical manipulators and study its convergence properties.

The control law will only require measurement of q, $\dot{q}$, and $\overline{F}$ and will not require *a priori* knowledge of the inertial parameters in $D(q)$, $C(q, \dot{q})$, and g.

The adaptive control law is obtained from (41) by replacing θ by their estimates $\hat{\theta}$, i.e.,

$$\tau = \tau_e + Y(r, \overline{F}, q, \dot{q})\hat{\theta} \quad (42)$$

which can also be rewritten as [see (41)]

$$\tau = \tau_e + Y\hat{\theta} + Y\theta - Y\theta = \tau_{\text{ideal}} + Y\tilde{\theta} \quad (43)$$

where

$$\tilde{\theta} = \hat{\theta} - \theta. \quad (44)$$

Introducing (43) in (30) and following a procedure similar to the one used in the Proof of Lemma 1 we obtain

$$D\dot{r} + Cr + Gr = Y\tilde{\theta} \quad (45)$$

with r as in (33). We can now propose the following parameter adaptation law:

$$\dot{\hat{\theta}} = -\Gamma Y^T r \quad (46)$$

where $\Gamma = \Gamma^T > 0$ is a constant matrix. This adaptation law together with (45) allows for global convergence of the algorithm as stated in the following lemma.

Lemma 3: Consider (45) and (46) with r as in (33)–(39). Then it follows that $r \in L_2 \cap L_\infty, \dot{q} \in L_\infty, \dot{r} \in L_\infty$ which implies that $r \to 0$ and finally $\tilde{F}$, $K^-\tilde{x}$, and $J^-\tilde{q}$ converge to zero as $t \to \infty$ and $\dot{\tilde{F}}$ and $K^-\dot{\tilde{x}}$ are in L_2. ▽

The proof is given in Appendix B.

Remark 3: The adaptive force/position control scheme presented in the previous section requires *a priori* knowledge of the stiffness matrix K in (8).

When K is unknown, the main problem is to propose an estimation algorithm that provides an estimate $\hat{K}$ such that: i) the equation error $F_t - \hat{K}(x_t - x_{e_t})$ converges to zero and ii) the estimate $\hat{K}$ converges to a constant full-rank matrix. If these two conditions are verified, the stiffness matrix K can be replaced by its estimate $\hat{K}$ in the adaptive scheme of the previous section. Convergence of the equation error in the L_2 sense is a standard property of either gradient or LS estimation algorithms. On the other hand, the estimate $\hat{K}$ can be constrained to be full-rank matrix by using the technique developed in [20].

Even though this seems to be a promising technique to cope with the problem of an unknown stiffness matrix, a complete stability analysis is not yet available.

Remark 4: Equations (3)–(6) and the decomposition in (24) and (25) allow us to avoid any force derivative measurement. However, this avoidance is obtained at the expense of some complexity in the control law, which is of crucial importance in view of a practical implementation.

V. Conclusions

This note has proposed an adaptive force/position control scheme for robot manipulators. The hybrid force/position controller is based on a particular decomposition of the robot Jacobian and environment stiffness matrices. The approach is applicable to general redundant robots working on a homogeneous environment. The control scheme does not require measurement of the joint acceleration nor the force first derivative.

Global convergence results are obtained in the sense that all signals remain bounded and the force and position tracking errors converge to zero.

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Appendix A

Proof of Lemma 1: Introducing (31) in (30) one obtains

$$D\left\{J^+R^{-1}\left[K^+\left(\ddot{F}_t - \dot{v}_f\right) + K^-(\ddot{x}_t - \dot{v}_x)\right] + J^-\ddot{\tilde{q}} + z\right\}$$
$$+ C\left\{J^+R^{-1}\left[K^+\left(\dot{F}_t - v_f\right) + K^-(\dot{x}_t - v_x)\right] + J^-\dot{\tilde{q}}\right\}$$
$$+ Gr = 0. \quad (A.1)$$

Taken now into account (32) and (33), (A.1) can be rewritten as

$$D\dot{r} + Cr + Gr = 0. \tag{A.2}$$

Using the results in Appendix B for $\theta = 0$, we obtain $r \in L_2 \cap L_\infty$. We will next prove that this implies that $\dot{q}$ is bounded.

Note that using (8), r in (33) can be rewritten as

$$\begin{aligned} r &= J^+R^{-1}[K^+(K\dot{x}_t - v_f) + K^-(\dot{x}_t - v_x)] + J^-\dot{\tilde{q}} \\ &= J^+R^{-1}[\dot{x}_t - K^+v_f - K^-v_x] + J^-\dot{\tilde{q}} \text{ [using (10)]} \\ &= J^+\dot{x} + J^+R^{-1}[K^+v_f - K^-v_x] + J^-\dot{\tilde{q}} \text{ [using (22)]} \\ &= J^+J\dot{q} + J^-\dot{q} - J^+R^{-1}\left[K^+v_f + K^-v_x - \dot{R}x\right] \\ &\quad - J^-\dot{q}_d \text{ [using (3)]} \\ &= \dot{q} - J^+R^{-1}[K^+v_f + K^-v_x] - J^-\dot{q}_d \text{ [using (9)]}. \end{aligned} \tag{A.3}$$

Due to physical limitations, (we consider revolute joints only) x is bounded and therefore F [see (8)], v_x and v_f [see (34) and (35)] are bounded too. Finally, it is clear from (A.3) that $\dot{q} \in L_\infty$. In view of P1) in Section II, $D^{-1}(q)$ is bounded, $C(q, \dot{q}) \in L_\infty$ and thus $\dot{r} \in L_\infty$. This implies (see Appendix C) that $r \to 0$ as $t \to \infty$.

Note that *KRJ* is bounded and then $KRJr \in L_2 \cap L_\infty$. Therefore, multiplying (33) by *KRJ* and using (17)–(20) one obtains

$$\dot{F}_t - v_f \to 0$$

and

$$\left(\dot{F}_t - v_f\right) \in L_2 \cap L_\infty. \tag{A.4}$$

Multiplying (33) by *RJ* and using (17) through (20) and (A.4) one gets

$$K^-(\dot{x}_t - v_x) \to 0$$

and

$$K^-(\dot{x}_t - v_x) \in L_2 \cap L_\infty. \tag{A.5}$$

Then, from (33), (A.4), (A.5) it follows

$$J^-\dot{\tilde{q}} \to 0$$

and

$$J^-\dot{\tilde{q}} \in L_2 \cap L_\infty. \tag{A.6}$$

Using (34) and (35) one obtains

$$\dot{F}_t - v_f = \left(\frac{d}{dt} + \lambda\right)\tilde{F} \tag{A.7}$$

$$K^-(\dot{x}_t - v_x) = \left(\frac{d}{dt} + \lambda\right)K^-\tilde{x}. \tag{A.8}$$

From (A.4) and (A.7), (A.5) and (A.8), and the results in Appendix C we finally conclude that $\tilde{F} \in L_2 \cap L_\infty$, $\tilde{F} \to 0$, $\dot{\tilde{F}} \in L_2 \cap L_\infty$, $K^-\tilde{x} \in L_2 \cap L_\infty$, $K^-\tilde{x} \to 0$, and $K^-\dot{\tilde{x}} \in L_2 \cap L_\infty$. ▽

Appendix B

Proof of Lemma 3: Consider the positive definite function

$$V = \frac{1}{2}r^TDr + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}. \tag{B.1}$$

Therefore

$$\begin{aligned} \dot{V} &= r^TD\dot{r} + 1/2r^T\dot{D}r + \dot{\tilde{\theta}}^T\Gamma^{-1}\tilde{\theta} \\ &= r^T[Y\tilde{\theta} - Cr - Gr] + 1/2r^T\dot{D}r \\ &\quad - \tilde{\theta}^TY^Tr \text{ [using (44)–(46)]} \\ &= -r^TGr \text{ (using property P3)}. \end{aligned} \tag{B.2}$$

From (B.2) and since $G = G^T > 0$, it follows that $r \in L_2$ and then $V \in L_\infty$ which implies that $r \in L_\infty$ and $\tilde{\theta} \in L_\infty$. The rest of the proof follows as in Appendix A.

Appendix C

Review of some results from L-theory are as follows:

1) We say that $r \in L_2$ if

$$\int_0^\infty r^Tr\,dt < \infty$$

$r \in L_\infty$ if r is bounded.

2) If $r \in L_2$ and $\dot{r} \in L_\infty$ then $r \to 0$ [23, page 232].

3) Let $H(s)$ be an asymptotically stable strictly proper transfer function. Then if $Y(s) = H(s)U(s)$ and $u(t) \in L_2 \cap L_\infty$, they $y(t) \in L_2 \cap L_\infty$, $\dot{y}(t) \in L_2 \cap L_\infty$ and $y \to 0$ [23, p. 59].

Appendix D

Proof of Lemma 2: Note that τ_{ideal} in (31) is a function of $\dot{v}_f$, $\dot{v}_x$, q, $\dot{q}$, v_f, v_x, z, and r. Using (34)–(38), (5), and (3)–(7) it is clear that v_f, v_x, $\dot{v}_f$, and $\dot{v}_x$ can be expressed as functions of q, $\dot{q}$, and $\tilde{F}$.

On the other hand, $\dot{F}_t$ can be expressed as a function of q and $\dot{q}$ using (3)–(7). Finally, it is known [24], [25] that $D(q)$, $C(q, q)$, and g in (1) are linear with respect to the inertial parameters that multiply known functions of q and $\dot{q}$. In view of this property and the fact that K is a constant matrix and $f(q)$, J, J^-, J^+, R and their derivatives are known functions of q, then τ_{ideal} in (31) can be rewritten as in (41) where θ contains the inertial and stiffness parameters and Y is a known matrix function.

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A Method for Verifying Sector Conditions in Nonlinear Discrete-Time Control Systems

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Abstract—**This note is concerned with the stability of a discrete feedback system with a continuous broken-line nonlinearity. Many nonlinearities in practice are included in this class. Sufficient conditions for this type of nonlinear feedback system to be asymptotically stable in the large are developed using the contraction mapping theory. It is then shown that Kalman's conjecture for a feedback system, with a sector nonlinearity also applies to discrete systems under a restricted sector condition. Another contribution of this note is to present a method for obtaining this restricted sector by solving a quadratic equation.**

I. Introduction

Although control systems often contain nonlinearities such as dead zone or saturation, a practical and meaningful stability theory for these systems is still lacking. The describing function [1] is a useful and practical method for studying the stability of such systems, but it gives only approximate results. Other methods, such as Popov's theorem, the circle criteria or the hyperstability theorem may be also applied to the stability analysis of such systems if they are regarded as a Lurie problem [2]. However, because these theories are concerned with a wide class of nonlinear feedback systems, the results are too conservative for the stability analysis of practical nonlinear systems.

In this note, the stability of a discrete feedback system with a continuous broken line nonlinearity is considered. Using the contraction mapping theorem, sufficient conditions for this type of nonlinear feedback system to be asymptotically stable in the large are derived. It is also shown that Kalman's conjecture for a feedback system with a sector nonlinearity also applies for discrete systems in the restricted form.

The sufficient conditions for stability of these nonlinear systems require the feedback nonlinearities to satisfy a certain type of sector condition. It is important to know the size of this sector in order to evaluate the stability region. It is shown that this problem is equivalent to finding a set of linear output feedback gains as a solution of the simultaneous Lyapunov problem which has been investigated in [5] and [6] for continuous-time systems. This note offers a simple calculation method for obtaining a stability region for the above nonlinear feedback systems by solving the discrete simultaneous Lyapunov problem. However, it should be noted that any stability region thus obtained is not necessarily the maximum set of the stability region.

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II. Sufficient Condition for Stability of Nonlinear Feedback Systems

Consider the single-input, single-output discrete nonlinear feedback system shown in Fig. 1 which consists of linear dynamic elements in the forward path and a memoryless nonlinearity $f(y)$ in the feedback path, described by the following equations.

A. Nonlinear Feedback System S

Linear Part:

$$x(t+1) = Ax(t) + bu(t), \qquad u(t) = -f(y(t))$$
$$y(t) = c^T x(t) \tag{1}$$

Nonlinear part:

$$f(y) = \begin{cases} k_1 y - (k_1 - k_2)\alpha & \text{if } y > \alpha \\ k_2 y & \text{if } -\beta \le y \le \alpha \\ k_3 y + (k_3 - k_2)\beta & \text{if } y < -\beta \end{cases} \tag{2}$$

where $x(t), b, c \in R^n$, $y(t), u(t) \in R^1$, $A \in R^{n \times n}$, and $\alpha, \beta > 0$. R^n and $R^{n \times n}$ denote the n-dimensional real vector space and $n \times n$ real matrix space, respectively.

Definition 1: Consider a linear system $\Sigma(A, b, c)$ described by (1). A set of real numbers $\Omega(P)$ is called a set of equivalent stabilizing feedback gains (ESF-gains) if there exists a positive definite matrix $P(=P^T)$ such that $(A - bc^T k)^T P(A - bc^T k) - P$ is negative definite for all $k \in \Omega(P)$.

This means that $x(t)^T Px(t)$ can be taken as a Lyapunov function for a system described by $x(t+1) = (A - bc^T k)x(t)$ for all $k \in \Omega(P)$.

Theorem 1: The origin of the nonlinear feedback system S is asymptotically stable in the large if k_1, k_2, and k_3 are ESF-gains.

Proof: Since k_1, k_2, and k_3 are ESF-gains, there exist symmetric and positive definite matrices, P, Q_1, Q_2, and Q_3, such that

$$(A - bc^T k_i)^T P(A - bc^T k_i) - P = -Q_i \qquad (i = 1,2,3). \tag{3}$$