Nonsmooth Mechanics, Springer Verlag London, Second edition, 1999: Erratum and addendum

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- In remark B.3, a.e. means almost everywhere in the Lebesgue measure.
- The appendix on dissipative systems has been dropped in the second edition, so the words about passivity in the introduction of appendix A have to be dropped.
- In appendix D, the fact that $0 \in \partial f(x_0) \Leftrightarrow x_0 \in \partial g(0)$... does not mean that g is differentiable, but sub-differentiable at $y = 0$.
- After remark 8.8: the constraint is implicitly assumed to be $q_1 \geq 0$.
- (8.102) reads $y + l_1 \sin(\theta) \geq 0$, $u + l_1 \lambda \cos(\theta)$, $\ddot{\theta}_3 = l_1 \lambda \cos(\theta)$.
- page 78: despite the fact.
- In (8.107), replace $\sin^2$ by $\cos^2$.
- page 148: see [854].
- page 142: Darboux derived these expressions...
- page 259: a bracket [is missing line 10 after max.
- page 321, line 17: drop the parantheses around t_∞.
- page 82, line 12: $\left(\frac{d}{dt}\nabla f(q)^T\right)\dot{q}$
- page 233, line 5: $f(q) \leq 0$
- page 220, line 1 after (5.111): in the unconstrained mode...
- page 485: the proximation corresponding to $f(\cdot)$ is $\text{prox}(z|f)$ or $\text{prox}_f z = \inf\{f(x) + \frac{1}{2}||z - x||^2\}$. Clearly if $f = \psi_K$ then $\text{prox}_f z$ is the point of K the closest to z.

- page 366, line 22: many more
- In (5.98) replace $\preceq$ by $\succeq$ and in the next lines also.
- Definition 1.8: in χ_{con} and χ_{rel}, it is $C\varphi(\cdot,\tau_0,x_0,u) > 0$ and $C\varphi(\cdot,\tau_0,x_0,u) < 0$.
- page 115, line -4: replace [159] by [164]
- page 112, Farka's lemma: $v^T\nabla f \leq 0 \Rightarrow v^T J^T n \leq 0$, see the book of Rockafellar page 200.
- page 328, line 4: (5.136)
- page 214: actually the result of uniqueness of solutions has also been shown for mechanical systems by Ltstedt in his 1982 paper (theoerm 5.4).
- page 134: collisions.
- page 169, line -14: 4.7 (2)
- Figure 5.19: $b < 0$
- Drop the footnote (7) page 191.
- In (5.53): $m(q)du - Q(t,q,u)dt = dP$.
- page 197, line 5: (10))
- In (5.84): $F_{1,t}$ in boldface
- In (7.36) (7.34): replace e by e_n
- page 490, line 5: then $y \in I$...
- page 336, line 5 of paragraph 6.6.1: "compare this with (6.17) for..."
- Figure 1.5: "attaining Ker(C)"
- Top of page 83: the problem under consideration is to minimize $G(x) = \frac{1}{2}[x+z]^T M^{-1}[x+z]$ under the constraint in (3.15), i.e. $x \in \mathcal{C}$. From the developments in (D.11) (D.12), $\text{argmin}_{x\in\mathcal{C}} G(x) = \text{proj}_{\mathcal{C}} z = \text{prox}_{\psi_{\mathcal{C}}} z$.
- In appendix B: notice that the contingent cone is not always convex, contrary to the tangent cone, see figure (D.1)
- page 487, line -14: $\partial\psi_{V(q)}(x) = \{y : ... - \langle x,y\rangle = 0\}$
- Change [159] to [161] pages 117 and 313.
- Top of page 71, change e to e_n.
- Bottom of page 71: change $f+\mu$ to $Q+\mu$.
- page 66, line 6: $\geq (F-\ddot{q})(v-q)$
- page 39: notice that λ_2 in (1.48) cannot render the set $q_1 \geq 0$ invariant (λ_2 taken as a measure), whereas λ_1 in (1.49) can do the job (together with a collision map). Therefore a major discrepancy between DAEs and systems with complementarity conditions.
- page 276: recall that $\dot{q}_{\text{norm}}$ is the component of $\dot{q}_n$ along $\mathbf{n}_q$, and $\dot{q}_{\text{tang}}$ is the component along $\mathbf{t}_q$, of the generalized transformed velocity (rather the transformed momentum), i.e. $\dot{q} = \dot{q}_{\text{norm}}\mathbf{n}_q + \dot{q}_{\text{tang}}\mathbf{t}_q = \dot{q}_n + \dot{q}_t$.
- page 99, line 16: ....[891], made use...

- page 103, line -1: requires
- page 339, change lines 17-18 by: ...$\in [0,1)$. As we shall see the incorporation of trajectory tracking during general robotic tasks will render it quite difficult to have both conditions.....Indeed the positive definite function $V(x)$ will not always match with the energy of the system. See chapter 8 and (8.46) for more details.
- page 428 after (8.46): notice that $\dot{V}(q,\dot{q})$ is not guaranteed to be sign definite on (t_k, t_{k+1}) due to the terms $-F_d\dot{q}(t)$ and $c(\lambda+c)q(t)\dot{q}(t)$.
- $\dot{y}_2^\star(k+1)$ in (8.114) stands for $\dot{y}_2^\star(t_{k+1}^-)$, the pre-impact robot desired velocity. Whence $\dot{y}_1(k) \triangleq \dot{y}_1(t_k^+) = \frac{m-e}{1+m}\dot{y}_1(t_k^-) + \frac{1+e}{1+m}\dot{y}_2^\star(k)$, implying that $\dot{y}_2^\star(k+1)$ is a function of $\dot{y}_2^\star(k)$.
- In remark 1.7: notice that outwards the constraint surface means inwards Φ, and inwards the constraint surface means outwards Φ.
- remark 5.14: notice that it is question here of smooth motion on small enough time intervals.
- page 339, line 12, section 7.1.2: we mean here that one should define a function V satisfying $\alpha(||.||) \le V(\cdot)$ without considering the unilaterality, which for instance precludes the use of potential energy $U(x) = mgx$. this is because we seek a function that may be used for the stabilization of complete robotic tasks, and has therefore to be suitable during free-motion tasks, in particular.
- page 375, line 13: suddenly
- page 377, line -12: losing; line -1: suddenly
- page 380, remark 7.17: suddenly
- page 321: these conclusions
- theorem 7.3: $(\dot{q}_k, \eta_k) \in \Sigma^+$
- Notice that (5.165) is a fixed point problem with unknown q_{k+1}. It can be shown that this fixed point problem has a unique solution (contraction argument). Moreover in some cases the formulation in (5.165) hides a much easier expression as the one-dimensional case shows.
- in (5.164): $h^2\partial\psi_{(1+e_n\Phi}(q_{k+1} + e_n q_{k-1}) \ni$ ...
- (5.83): $-V_{A_1}(t_k^+) \in \mathrm{proj}_{\mathcal{T}}\partial\psi_{\mathcal{C}}(F'_{1,\mu})$
- page 321, line -2: in a V-form...
- page 320: if $e_1 = e_2 = 1$, then there is a finite multiplicity, see [675] page 5-6 chapter 4. this is intuitively sounds since there is no energy loss in this case, so a finite accumulation cannot occur in an angle in dimension 2.
- there is a link between the rule $\dot{q}(t_k^+)$ =$\mathrm{prox}_{M(q)}[V(q), \dot{q}(t_k^-)]$, and the basis $(\mathbf{n}_q, \mathbf{t}_q)$. $\mathrm{prox}_{M(q)}[V(q), \dot{q}(t_k^-)] = \dot{q}_t = \dot{q} - \dot{q}_n = \dot{q} - (\dot{q}^T M(q)\mathbf{n}_q)\mathbf{n}_q$.
- page 146: $F_n dt = m d\dot{y}$ or $F_n dt = \frac{m\rho^2}{\rho^2+x^2} dv_{r,n}$
- page 30, middle of the page: relies at each step on the fact that there is a unique...
- A result by Ballard shows that remark 2.11 should be modified: he exhibited a case with plastic collision and non-unique solution, in the spirit of Bressan and Schatzman counter-examples. The trick to assure uniquenss is to imose all data to be analytic.

  P. Ballard, 1999 "Well-posedness of the dynamics of discrete mechanical systems with perfect unilateral constraints", submitted in Archive for Rational Mechanics and Analysis.

- About Painlevé problem: Klein and Prandtl used a penalization to examine this problem.
- page 390: the result in [891] can be used to state the asymptotic stability of (7.73) and of P_Σ, i.e. $\dot{x}(t_k^+) \to 0$ as $k \to +\infty$.
- page 265, line 6: drop "task-space", replace by "a particular frame".
- page 73 line 11: $q_n \to q$, not xq.
- In problem 5.2, we have used the equivalence between $N_{C(q(t))}(R_\mu^{'}(t))$ and $\partial\psi_{C(q(t))}(R_\mu^{'}(t))$ as in (5.81) or (5.83). Notice that the notation $C(q(t))$ highlights the fact that one works in the configuration space.
- (5.85) uses the fact that $(u^+ - u^-)\delta_{t_k} = dR$ at a shock and t_k is an atom for the measure dP also at that time, which is also an atom of the positive measure $d\mu$. Since $N_C(\cdot)$ is a cone, one can replace $R_\mu^{'} = \frac{dR}{d\mu}$ in $N_c(R_\mu^{'})$ by $N_C(u^+ - u^-)$ since this just amounts to multiplyng the argument by a scalar.
- Suppress [216] which is the same as [209]
- page 20, line 12: change poses by creates
- page 257: Moreau uses a midpoint Euler [369] page 517. For more details on numerical simulations in complementarity systems, see B. Brogliato, A.A. ten Dam, L. Paoli, F. Génot, M. Abadie, "Numerical simulation of finite dimensional multibody nonsmooth mechanical systems", ASME Applied Mechanics Reviews, 2002.
- page 206, after (5.81): figure 5.13 instead of 5.3.4
- page 139,line 7: n_1, t_{11}, t_{12} should be in boldface, and also in (4.79) (4.80) inside the integral sign. Also page 114 lines -3, -7, and page 143 lines 1 and 3.
- page 139, before (4.79), one implicitly assumes that $\mathbf{n}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{t}_{11} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{t}_{12} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.
- page 140, remark 4.12: experiences should be experiments
- page 142, line -8: Darboux derived these expressions. In (4.96): $\frac{1}{h(\mu,\zeta')}$
- page 143, in (4.100): now from (4.85) (4.91) and (4.99) one has:
- page 14: the simplest MDE as in (1.23) is $\dot{x} = x\dot{u}$. When u is discontinuous at t then this writing is meaningless at t. One has to disregard this differential equation at such times and give a meaning to the solution outside t. This can be done in such simple case easily by rewriting it as $\frac{\dot{x}}{x} = \dot{u}$. Then the function $y \triangleq \ln(x)$ jumps at t, so that $\dot{y}$ is a Dirac measure at t. Notice that these are equalities of distributions, or measures. Then it is possible that the solution of a system à la Bainov and the solution of ODEs as in (1.23) coincides. But they are basically of quite different nature. The study of (1.23) is definitely much more difficult and one has to resort to advanced mathematical studies to give a meaning to such ODEs. For instance the paper M. di Paola, A. Pirrotta, Non-Linear Mechanics, vol.34, pp.843-851, 1999, actually implicitly defines a jump function as in systems (7.1), and shows that for the simple linear case this provides the same solution as using the change of variable above.
- page 303: Since we choose kinematic coefficients, it is not possible that the e_is are associated to a pari of bodies (since Newton's law becomes useless when $v_{r,n}(t_k^-) = 0$). But this may not be the case if Poisson or energetical coefficients are chosen, together with a Darboux-Keller model.

- page 161, line 10: this is natural since Darboux-Keller assumes no tangential compliance, i.e. $k_x = 0$.
- footnote 33 page 137: ...is also sometimes called...
- page 146: line 1: $\begin{pmatrix} a & h^T \\ h & B \end{pmatrix}$.
- The metric tensor for tangent vectors to the configuration space is $M(q)$. The metric tensor for cotangent vectors is $M^{-1}(q)$. This is why orthogonality is expressed as $\nabla f_i^T M^{-1}(q) \nabla f_j = 0$ or equivalently $\mathbf{n}_{q,i}^T M(q) \mathbf{n}_{q,j} = 0$. Indeed $\nabla f_i \in T_q^\star Q$ is a covector (like the generalized momentum $p = M(q)\dot{q}$) while $\mathbf{n}_q \in T_q Q$ is a vector.
- page 77, line 3, §3.1: mass with zero initial velocity.
- page 83, line 7 §3.2.1: same well-known free-motion.
- page 79, line 4 after (3.5): $[...]^T M^{-1}(q)[...]$ is minimum. Indeed this can be easily seen since time, positions and velocities are considered fixed (in other words ifone computes the derivative of this expression then all terms $dt, dq, d\dot{q}$ vanish and only $d\ddot{q}$ remains). Actually this is closely related to Gauss'principle, see section 3.2.
- page 79, line -2: ...permits this. In fact the whole trick is to determine $\ddot{q}(\tau_0^+)$, knowing that $q \equiv \dot{q} \equiv 0$ on $[\tau_0 - \epsilon, \tau_0)$. We shall come back on this fundamental problem in sections 3.2 and 5.4.
- About multiple shocks: the notions of multiplicity (the codimension of the striked singularity of $\partial\Phi$) and the order O of a shock (the number of collisions) should be properly defined. They are quite different.
- page 368: when the string is loose
- page 324, line 14: ....in chapter 5 and section 6.5.8.
- page 95, line -3 before remark 3.3: $\mathbf{n}_q$
- page 269, line 1 after 6.2.1: no transpose.
- Footnote page 193: according to Glocker, ZAMM vol.78, 1998, the part μ_{na} of dR might be a consequence of fractal interaction forces. In the same paper Glocker discusses the various virtual work principles in the setting of complementary systems (no impact).
- (5.53): $M(q)du - Q(...)$..
- After (5.51): we write $-Q(t, q, \dot{q})$ to shorten...
- page 397, line 8: ...feedback control of a class...
- page 256: Ltstedt used a multistep method.
- page 424, line -11: ...dynamics is...
- suppress [651]
- reference [795]: Springer Verlag London, CCES.
- page 78,line -10: despite the fact
- page 482, line 8 after (D.3): unilaterally
- In (6.156): parantheses missing in ... $+ I\dot{\theta}(t_k^-))IB(\theta, \mu)$
- page 338, line 5: looks like

- page 197, line 5: ...each q)
- page 261, line 15: similar
- page 258, line -12: tangential
- page 137, line -1 before (4.71): $\frac{dp(t')}{dt'}$
- page 155, (4.126): $\frac{T(t_f)-T_m}{T(0)-T_m}$
- page 216, last line: $M = \nabla f^T M^{-1}(q) \nabla f$
- page 311, line 1: there is an infinity
- page 372 and the following: the unilateral constraint should be $f(q) = q - q_{\min}$ instead of $q - q_0$. So modify (7.54), (7.55), (7.56), (7.58) in accordance. This does not change the conclusions on grazing bifurcations.
- page 144, line 3 after (4.101): $\zeta(p_{n,e}) = ...$)
- page 311: suddain should be sudden
- page 310: permitting
- footnote 22 page 314: this is OK but not in 2D. At least in 3D since at the end the "particle" mat "slide" along $\partial\Phi$.
- page 183: "To the best of our knowledge, the only way ...contact. In other words, introduce more physical information in the model, and convert it into a rigid body approach through the definition of suitable coefficients which allow one to predict impact outcomes.
- Some authors (J. Garcia de Jalon, E. Bayo, 1994 *Kinematic and dynamic simulation of multibody systems. The real-time challenge*, Springer Verlag, Mechanical Engineering Series.) call exogeneous impacts "percussions", and shocks between bodies "impacts". We do not adopt this terminology here.
- page 291, line 5 in 6.5.1: (by...
- page 291, line 7: $\mathbf{n}_q$
- remark 6.19: coefficients
- footnote 12 page 201: $f(\cdot) \in C^{1,\alpha}$ means that...
- page 214, lines 12-15: equivalent in the sense that a solution exists and is unique for one system if and only if it exists and is unique for the other one.
- page 58, line -1: [159] should be [161]
- page 331, line 6 after (6.161): see remark 4.7
- page 313, remark 6.26: change [159] by [161][164]
- Notice in section 6.5.4 pages 304-307 that our solution incorporates not only coefficients e_i that respect constraints ii), iii), iv) in **general algorithm**, but also the ratio α. This indicates that restitution coefficients are in general not sufficient to describe a multiple impact. further coefficients are needed. this is in agreement with a recent work by Hurmuzlu: Y. Hurmuzlu, V. Ceanga, 2000 "Impulse correlation ratio in solving multiple impact problems",In *Impacts in Mechanical Systems. Analysis and modeling*, Springer Verlag, Berlin, Lecture Notes in Physics, B. Brogliato (Ed.).

- page 330, line -1: $f(\theta)=0$ for central impacts
- About the multiple impact rule in (3.21): apparently this was formulated first by Carnot in 1803.
- About the spring-dashpot model: notice that this model does not even guarantee that the contact force remains ≥ 0 during the contact phase. However its impulsion does, and this is what counts for us in this developments.
- page 55, line 9: well-known
- page 36, last line: see section 8.1
- page 68: $t_{k+1} > t_k + \delta$, δ depends on initial data.
- page 174, last §: $f_i^{(j)}(q(t_0)=0$
- example 5.3: this is related to the pointedness of the cone formed by the sum of the friction cones, in the configuration space (generalized cone). In the case in question here, the cone is not pointed. The jamming conditions are $\mu >$Arctan(α), where α is the half of the angle in which the disc is wedged. In our case we have treated $\alpha=\frac{\pi}{4}$ so the "critical" value for μ is 1, as indicated.
- (6.72): $f_2(q)=b^{'}=y-\frac{l}{2}\cos(\theta)+\frac{L}{2}\sin(\theta)\geq 0$.
- page 204, line -3 in remark 5.12: [704]
- Notice that any piecewise linear characteristic can be transformed in a corner law by linear transformation.
- page 102, line -9: which allows
- page 100, line 6: drop "recently"
- page 84, (3.20): $\frac{1}{2}\lambda^T\nabla f^T M^{-1}\nabla f\lambda+\lambda^T\nabla f^T\dot{q}$...
- remark 4.18, see [162]
- pages 408-409: $\dot{q}_r(t_k^+)=\nabla_{\tilde{q}}f_t^T\dot{\tilde{q}}(t_k^+)+\nabla_t f_t=-e_n\dot{q}_r(t_k^-)$. Notice that $\dot{q}_r(t_k^+)\in V_t(\tilde{q})$ while $\dot{q}_r(t_k^-)\in -V_t(\tilde{q})$. Also $\dot{q}_r=\nabla_q f^T\dot{q}$. So since $\dot{q}(t_k^+)$ satisfies $\nabla_t f_t+\dot{q}(t_k^+)^T\nabla_q f\geq 0$, it follows that $\dot{q}_r(t_k^+)+\nabla_t f_t\geq 0$. Thus if $V_t(\tilde{q})=\{v:\nabla_{\tilde{q}}f_t^T v+\nabla_t f_t\geq 0\}$, one has $\dot{\tilde{q}}(t_k^+)\in V_t(\tilde{q})$, i.e. $\dot{q}_r(t_k^+)\geq 0$, $\dot{\tilde{q}}(t_k^-)\in -V_t(\tilde{q})$, i.e. $\dot{q}_r(t_k^-)\leq 0$.

  Let us note that these developments are different from those in J.J. Moreau, 1999 "Some basics of unilateral dynamics", Proc. of IUTAM Symposium on Unilateral Multibody Contacts, Kluwer, F. Pfeiffer, Ch. Glocker (Eds.), pp.1-14, because therein one considers $f(q,t)$ whereas we have $f(\tilde{q},t)$.
- page 235, in front of (5.137): $-\frac{(\dot{\theta}_{c1}^{\pm})^2}{m}$
- page 377, line -12: losing

  page 206, line -1 before (5.82): figure 5.13
- Footnote (15) page 207: in appendix D, not in chapter A.
- In (5.63): $\partial\psi_{V(x)}(u(t^+)$
- page 198, line 2: similar modification
- (5.58): $N_{V(q(t))}(u(t^+)$, otherwise the sweeping process formulation does not encompass discontinuous velocity instants.

- About cones: in general one draws the sets $x + S(x)$ rather than $S(x)$ (normal, tangent, contingent, friction cones).
- page 79, line 4 after (3.5): $[Q - ...]^T M^{-1}(q)[Q - ...]$
- Example D.1: actually $\partial f(0)$ is just equal to $[\dot{f}(0^-), \dot{f}(0^+)]$, see Rockafellar's book page 72.
- In definition D.2, the overbar means the closure of the set, not the complement. So definitions D.2 and 5.1 are equivelent (at least as long as the considered point is not outside the set). And the cones in figure D.1 have reversed signs.
- For a more general description on how to obtain the constraint $f(q) \geq 0$ in chapter 4, see Ch. Glocker, 1999 "Formulation of spatial contact situations in rigid multibody systems", special issue of Computer Methods in Applied Mechanics and Engineering, on Computational Modeling of Contact and Friction, J.A.C. Martins, A. Klarbring (Eds.), vol.177, pp.199-214.
- The exact condition in theorem ?? in chapter 5, section on complementarity conditions, is: $\text{rank}_{\text{column}} \nabla\phi = \min(n, m)$. This avoids hyperstatic systems since this implies that $\nabla\phi$ is a row matrix. This full-column rank condition is used in many works since it does simplify the problem. However in practice most systems are hyperstatic and one has to resort to other tricks to calculate the multipliers (contact forces). Evidently in this case there is an infinity of solutions for the Lagrange multiplier. This may be bothering for users who wish to get the value of the contact forces. As is well-known, introducing penalizations at the contact classically solves this indetermination problem. However this is a procedure that should be used only when one desires to have an accurate idea of the contact forces, since penalizations will often imply stiff equations. Moreover, due to the difficulty one encounters when identifying contact models, even such a penalization procedure may not be so accurate, though it looks quite simple and natural to many.
- Some conclusions on the Painlevé-Klein example are wrong. In particular one does not have $m\ddot{x} = F + T$ because another term, taking in to account some inertial effects (namely the moment of the inertial force with respect to the point of application of the force F). Most probably some of the tools used to analyze the Painlevé example, presented in the same chapter, could be used in this example as well.
- In (5.12) this is evidently $\nabla h_q(q_0)\lambda$ and not $\lambda^T \nabla h_q(q_0)$, since ∇_q denotes the gradient, not the Jacobian.
- Footnote 15 page 69 contains erroneous assertions. First of all the solution is not unique for $q(0) = 0$, otherwise the basic conditions for uniqueness are satisfied. For this initial data there are three solutions: $q(t) \equiv 0$ and the other two as indicated in the footnote. Since continuity means that for any sequence $q_0^n \to q_0$ one has $\phi(t; t_0, q_0^n) \to \phi(t; t_0, q_0)$, clearly the solution is not continuous with respect to initial data at $q(0) = 0$ (though these functions are continuous functions of q_0!).