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Abstract

The present work provides a link between different simplified models of neuronal activity. In particular, we have shown how models of threshold type and generic bifurcation models are related. We expect that our approach will be appropriate for analytical network studies. We also develop a simple and general method to predict neuronal spike train using a threshold model. This technique provides a framework to study the integration of inputs in real neurons. Finally, by using a modelling approach at the level of simple conductance-based models and small neuron assemblies, we study the characteristic features of the individual dynamics of neurons and their cooperative behavior. In particular, the conditions for a single neuron to generate bursting oscillations and to react as a coincidence detector are established. Regular and small-world networks of such coupled systems are also investigated. It is shown that synchronization properties of dynamical neural networks essentially depend on the coupling configuration, the number of cells, and the type of coupling.

The project was divided into three principal tasks of increasing complexity. During the reported period we have been working mainly on the first and the third tasks. The first task concerning the single neuron models is almost finished. The third task, that is directed to the study of networks of neurons with fixed couplings, is being studied. Intensive study of this task will be started next year. The second task, dedicated to decoding of neuronal spike trains will also be started next year. So far, no major modifications in the work tasks have occured. Below, we describe the main results and the advancement of the project in detail.

Work package 1: Towards simple neuron models

1. **Analytical reduction to threshold models.** In this part of the project we have developed a technique for the analytical mapping between different models. We aim at elucidate the relations between these models.

   The time evolution of the neuronal activity is commonly describe by nonlinear differential equations. On the other hand, single variable threshold models have been proposed. We have developed a general technique for the mapping of differential neuronal models to threshold models. Because of the non linearity of the equations, it is often not possible to access to an analytical expression for the solutions. Thus, beyond numerical simulations, the comparison between the different formulations of the neuronal activity is a hard problem. Our approach consists in a piecewise linear reduction of the nonlinear functions. We restrict our reduction to a single line of discontinuity that allows an explicit representation of the threshold. Thus, the neuron presents two regimes which are called the subthreshold and superthreshold regime.

   In [1] we applied this technique to two-dimensional membrane models, namely the FitzHugh-Nagumo model and the Morris-Lecar model. Alternatively our two-dimensional models may be seen as a generalization of Integrate-and-Fire models. This generalization follows two steps (i) we introduce a superthreshold behavior and (ii) we consider a smooth recovery process using an additional variable. At each step the relation with the Spike Response Model is presented.

   The present work provides a link between different simplified models. In particular, we have shown how the threshold type and the generic bifurcation model type are related. Moreover we expect that our approach will be appropriate for analytical network studies.

   Advancement: This subtask is finished.

   Contributors: Arnaud Tonnelier and Wulfram Gerstner (LCN, EPFL)

2. **Predicting single neuron activity** In this part of the project, we have developed a simple method to map a generic threshold model, namely the Spike Response Model (SRM), to data of neuronal activity using a minimal amount of information. We have tested systematically to which extent it is possible to predict the full spike train of a neuron once the mapping of the SRM to the data is done.

   A method to map the SRM to data and predict the spike train of neurons has been successfully developed. Our method was first tested on simulated data (data were generated with a Hodgkin-Huxley-like model). The detailed model was stimulated with biologically plausible time-dependent current. We sampled the subspace of parameters by varying the mean current and it's standard deviation in a biologically relevant range. We observed that, once the mapping is realized for given characteristics of the current, it is very robust to variations in those ones. The SRM achieves very reliable prediction of the membrane voltage of the neuron in the subthreshold regime. Moreover, it also achieves prediction of up to 80% of the spikes with the correct timing within a small time window.

   Finally, we started to test our method on experimental data (*in-vitro* recordings of rat cortical cells during current injection). Preliminary results show that our method allows good prediction of experimental data as well. This work describes a simple and general method to predict the activity of real neurons in cortex. It also provides a framework to study the integration of inputs in real neurons.

   Advancement: This subtask is partly finished. During first quarter of year 2003, we will consider the effect of dendritic architecture and conductance-based input on the predictive power of the SRM.

   Contributors: Renaud Jolivet and Wulfram Gerstner (LCN, EPFL)

3. **Bursting neuron models.** As a main bursting neuron model, the Hindmarsh-Rose system was chosen. This model belongs to a class of Hodgkin-Huxley conductance-based models in which the complicated current-voltage relationship of the conductance based models are replaced by polynomials in the dynamical variables.

   Based on bifurcation theory, we studied basic scenarios leading to the emergence of bursting oscillations in the Hindmarsh-Rose model. We showed rigorously bifurcation diagrams and regions of parameters with complicated dynamics presenting different types of burstings.

   The Hindmarsh-Rose is one of the most popular low-dimensional neuron models exhibiting complicated dynamics. Therefore when modelling the cooperative behavior of bursting neurons, this model is often used as a unit. We will also use this model to execute Task 3 concerned with networks of coupled neuron oscillators.

   Advancement: This subtask is finished and will be submitted for publication soon.

   Contributors: Igor Belykh and Martin Hasler (LANOS, EPFL)

Work package 3: Synchronization phenomena (small networks)

1. **Single neuron as a coincidence detector.** We investigate the possibilities of modelling coincidence detection in various nonlinear dynamic neuron models. Two spike trains coming from two different neurons were applied to the input of a third neuron (the so-called slave neuron). The question is whether these models can be set up to react on the correlation between two incoming spike trains. Simulations were done for both spiking and bursting neuron models. Recently some promising results have been obtained in the simulations that indicate the possibility of subthreshold coincidence detection.

   Currently the focus in this part of the project is on two points. On one hand the usefulness of discrete model-maps is being investigated. Model maps imitate the behaviour of the continuous models in discrete time. The advantage is shorter computing times and more analytical insight. On the other hand the research is being directed to a more statistical approach to coincidence detection meaning that one looks at quantities such as spiking frequency and correlation between signals.

   Advancement: This subtask is partly finished and it will be continued during the first quarter of the year 2003.

   Contributors: Enno de Lange, Igor Belykh, and Martin Hasler (LANOS, EPFL)

2. **Synchronization and clusters in networks of locally linearly coupled systems.** Lattices of diffusively coupled oscillators are studied. First, the network of identical oscillators was considered and the symmetries of the resulting system of coupled oscillators were exploited. Depending in an essential way on the number of oscillators composing the network, the set of possible regimes of spatiotemporal synchronization is examined. Conditions of the stability of cluster synchronization are obtained analytically for a wide class of coupled dynamical systems with complicated individual behavior [2, 3, 4]. Second, the persistence of clusters in the presence of parameter mismatch between the systems is investigated. For this case, analytical and numerical studies allowed us to conclude that these cluster synchronization regimes persist against up to $10 - 15\%$ parameter mismatch and against small noise [5].

   This study contributes both to the general synchronization theory of coupled oscillators and to the synchronization theory of dynamical neural networks. In the last case, this type of linear coupling refers to electrical synaptic coupling that results in levelling of membrane potentials, i.e. in a decrease of the error signal between the neurons. Such a coupling plays a decisive role of the activity of different groups of neurons in some invertebrates (e.g. a leech).

   Advancement: This subtask is finished and the results are [in press] published [2, 3, 4, 5].

   Contributors: Igor Belykh and Martin Hasler (LANOS, EPFL)

3. **Synchronization in small-world networks.**

   The small-world networks are regular coupled lattices having a few additional randomly arising long-range shortcuts. In such networks the average distance between nodes comes down dramatically and accounts, for example, for such a phenomenon as short neuronal paths in the brain (many neural networks are shown to be small-world systems).

We proposed a new type of dynamical small-world networks with a time-varying coupling configuration. Rigorous and tight bounds on the strength of coupling between the cells have been established that are necessary to achieve complete synchronization independently of the initial conditions. The synchronization thresholds have been explicitly linked with the characteristic path length of the coupling graph and with the probability of shortcut appearance. It is proven for this new model that a few random shortcut additions lower significantly the synchronization threshold and an effective characteristic path length.

Advancement: This subtask is being finished for networks with linear coupling and will be submitted for publication soon. For small-world neural networks with chemical synaptic coupling (fast-threshold modulation type) it will start from the second quarter of the year 2003.

Contributors: Igor Belykh and Martin Hasler (LANOS, EPFL)

4. **Synchronization of bursting neurons coupled via fast-threshold modulation.**

   We studied analytically a network of bursting Hindmarsh-Rose models coupled via fast-threshold modulation (fast-threshold modulation is a model of chemical synaptical coupling). For this network and for excitatory synaptic coupling, we proved the stability of synchronization in the regime of bursting. Influence of parameter mismatch between the neurons as well as other types of synchrony are being investigated.

   Advancement: This subtask is being studied and will be ready for publication soon. It will be also continued during two first quarters of the year 2003.

   Contributors: Igor Belykh and Martin Hasler (LANOS, EPFL)

Important events

Within the project, we participated in the following international conferences and workshops:

- Int. Conference on Nonlinear Theory and its Applications (NOLTA-2001), Japan, 2001 [2].
- Int. Conference "Progress in Nonlinear Science dedicated to the 100-th anniversary of A.A. Andronov", Nizhny Novgorod, Russia, 2001 [3].
- Int. Workshop and Seminar "Control, Communication, and Synchronization in Chaotic Dynamical Systems", Dresden, Germany, 2001. Invited lecture: Igor Belykh "Invariant manifolds and chaotic synchronization".
- Int. Workshop "Synchronization, collective behavior and complex phenomena in chaotic systems", Florence, Italy, 2002. Talk: Igor Belykh "Synchronization in small-world networks with a time-varying coupling configuration".
- NATO Workshop "Synchronization: theory and application", Crimea, Ukraine, 2002. Talk: Igor Belykh "Cluster synchronization in lattices of coupled oscillators: symmetries, stability, and persistence".
- Int. Conference "BioComp 2002", Vietri sul Mare, Italy, 2002. Poster: Arnaud Tonnelier "Dynamical Behavior of the piecewise linear FitzHugh-Nagumo equations".

Publications within the project

[1] A. Tonnelier and W. Gerstner, Piecewise linear differential equations and new integrate-and-fire neurons: insights from 2-dimensional membrane models, *Accepted in PRE* (2003)

[2] I. Belykh, V. Belykh, K. Nevidin, and M. Hasler, "Spatiotemporal synchronization in three-dimensional lattices of coupled chaotic systems", Int. Conference on Nonlinear Theory and its Applications (NOLTA-2001) , Japan , Vol. 1, pp 89-92, 2001.

[3] I. Belykh, V. Belykh, K. Nevidin, M. Hasler, "Cluster synchronization in lattices of diffusively coupled dynamical systems", International Conference Progress in Nonlinear Science, Nizhny Novgorod, Russia, 2001.

[4] V. Belykh, I. Belykh, K. Nevidin, M. Hasler, "Cluster synchronization in three-dimensional lattices of diffusively coupled oscillators", Int. Journal of Bifurcation and Chaos, Vol. 13, N 4, 2003 (in press).

[5] I. Belykh, V. Belykh, K. Nevidin, and M. Hasler, "Persistent clusters in lattices of coupled non-identical chaotic systems", CHAOS, Vol. 13, N 1, 2003 (in press).