An algebraic approach to granularity in time representation

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Abstract

Any phenomenon can be seen under a more or less precise granularity, depending on the kind of details which are perceivable. This can be applied to time. A characteristic of abstract spaces such as the one used for representing time is their granularity independence, i.e. the fact that they have the same structure at different granularities. So, time "places" and their relationship can be seen under different granularities and they still behave like time places and relationship under each granularity. However, they do not remain exactly the same time places and relationship. Here is presented a pair of operators for converting (upward and downward) qualitative time relationship from one granularity to another. These operators are the only ones to satisfy a set of six constraints which characterize granularity changes.

1 . Introduction

"Imagine, you are biking in a flat countryside. At some distance ahead of you there is something still. You are just able to say (a) that a truck (T) is aside a house (H), it seems that they meet. When you come closer to them (b) you are able to distinguish a bumper (B) between them, and even closer (c), you can perceive the space between the bumper and the house."

This little story shows the description of the same reality perceived at several resolution levels: this is called granularity. Granularity would not be a problem if different individuals, institutions, etc. would use the same granularity. This is not the case and, moreover, these individuals communicate data expressed under different granularities. There could be a problem if, for instance, someone at position (a), asked "how would you call that which is between H and T?" because at that granularity, the description of the scene would assume that there is nothing between H and T. The study of granular knowledge representation thus tries to express how the same phenomenon can, in some sense, be consistently expressed in different manners under different granularities. This is achieved through operators which, for a situation expressed under a particular granularity, can predict how it is perceivable under another granularity.

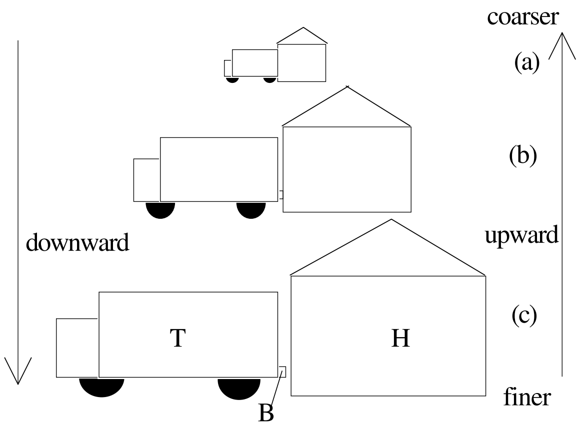

Figure 1. The same scene under three different granularities. This is taken as a spatial metaphor for time granularity and is used throughout the paper.

Granularity can be applied to the fusion of knowledge provided by sources of different resolution (for instance, agents — human or computers — communicating about the same situation) and to the structuring of reasoning by drawing inference at the right level of resolution (in the example of figure 1, the first granularity is informative enough for deciding that the truck driving wheel is on the left of the house — from the standpoint of the observer).

On one hand, in [10], granularity is expressed granularity between two, more or less detailed, logical theories. On the other hand, the physical time-space and its representation have been well-studied because many applications require them. A very popular way to deal with time is the representation of relationships between time intervals [2]. To our knowledge, qualitative time granularity has never been studied before. Jerry

in: Scott Goodwin, Howard Hamilton (éds.), Proceedings of 2nd international workshop on temporal representation and reasoning, Melbourne (FL US), pp147-154, 1995

Hobbs [10] introduced granularity in an abstract way (i.e. not connected to time) and [11, 4] introduced operators for quantitative time granularity which share a common ground with ours (see §7).

The paper first recalls some basics about time representation (§2). This section can be skipped by those who already know the subject. Then, the usual interpretations of time and granularity in this context are introduced. Afterwards, required properties for granularity change operators in the classical time algebra are presented (§3). This part is very important since, once accepted the remainder is directly deduced. The only set of operators (for instant and interval algebra) satisfying the required properties are thus deduced in §4. The results concerning the relationship between granularity and inference are then briefly presented (§5).

The proofs of all the propositions, but the "only" part of the first one, can be found in [6]. The main results (but those of §5) are from [7].

2. Background

Classical notions about temporal algebras, neighborhood structures and instant-interval conversions are presented here.

2.1. Temporal algebra

There has been considerable work carried out on qualitative time representation. We recall here several notions about the algebra of topological and vectorial relationships holding between time entities.

An instant is a durationless temporal entity (also called time point by analogy with a point on a line). It can be numerically represented by a date. Qualitatively representing these instants requires identifying them and putting them in relation. There are three possible mutually exclusive relationships between instants. They are called «before» (<), «after» (>) and «simultaneously» (=). The set {<, =, >} is called A_3.

relation (r): *x1* r *x2*	*x1*/*x2*	reciprocal: *x2* r^{-1}*x1*
before (<)		after (>)
simultaneously (=)		=

Table 1. The 3 relationships between instants *x1* and *x2*.

$\times_3$	>	=	<
>	>	>	< = >
=	>	=	<
<	< = >	<	<

Table 2. Composition table between instant relationships.

It is sometimes possible to deduce the relationship between two instants *x* and *z*, even if it has not been provided, by propagating the otherwise known relationships. For instance, if *x* is simultaneous ({=}) to *y* which is anterior ({<}) to *z*, then *x* is anterior to *z*; this is called composition of temporal relations. The composition operator $\times_3$ is represented by a composition table (table 2) which indeed indicates that $=\times_3<$ gives {<}.

A (continuous) period is a temporal entity with duration. It can be thought of as a segment on a straight line. A numerical representation of a period is an interval: a couple of bounds (beginning instant, ending instant) or a beginning instant and a duration. Intervals can be manipulated through a set of 13 mutually exclusive temporal relationships between two intervals (see table 3); this set is called A_{13}.

relation: *x1* r *x2*	*x1*/*x2*	reciprocal: *x2* r^{-1}*x1*
before (b)		after
during (d)		contains
overlaps (o)		overlapped by
starts (s) (and finishes before)		started by (and finishes after)
finishes (f) (and start after)		finished by (and starts before)
meets (m)		met by
equals (e)		e

Table 3. The 13 relationships between two intervals *x1* and *x2*.

The composition operator $\times_{13}$ is represented by a composition table [2], similar to the table 2, which allows to deduce, from a set of intervals and constraints between these intervals, the possible relations between any two of these intervals.

2.2. Extensions of notations

Let Γ be either A_{13} or A_3, $\vee$ be the logical disjunction and $\times$ be the composition operator on Γ, the following notations are used (in a general manner, $<2^\Gamma \subseteq \times>$ is an algebra of binary relationships). The lack of knowledge concerning the actual position of some temporal entity *x* with regard to the temporal entity *y* is expressed by a sub-set ρ of Γ which is interpreted as the disjunction of the relations in ρ:

$$x\rho y = \bigvee_{r\in\rho} xry$$

Thus, *x*{b m}*y* signifies that the temporal entity *x* is anterior to or meets the temporal entity *y*. The following conventions are used below:

- When a result is valid for both algebras, no distinction is made between the temporal entities concerned. The base sets (A_{13}, A_3, and maybe others), as well as the composition $\times$ and reciprocity $^{-1}$ operators are not distinguished;

- The letter ρ represents a sub-set of the corresponding base set of relations ($\rho \subseteq \Gamma$); the letter «r» represents a relationship.
- ρ^{-1} represents the set of relations reciprocal of those contained in ρ: $\{r^{-1}; r \in \rho\}$.
- $\rho_1 \times \rho_2$ represents the distribution of $\times$ on $\vee$:

$$\rho_1 \times \rho_2 = \bigcup_{r_1 \in \rho_1, r_2 \in \rho_2} r_1 \times r_2$$

2.3. Neighborhood structure

Two qualitative relations between two entities are called *conceptual neighbors* if they can be transformed into one another through continuous deformation of the entities [9]. A conceptual neighborhood is a set of relations whose elements constitute a connected sub-graph of the neighborhood graph.

DEFINITION (conceptual neighborhood): A conceptual neighbor relationship is a binary relation N_Γ^X on a set Γ of relations such that $N_\Gamma^X(r_1,r_2)$ if and only if the continuous transformation of an entity o_1 in relationship r_1 with another entity o_2 can put them in relation r_2 without transition through another relation.

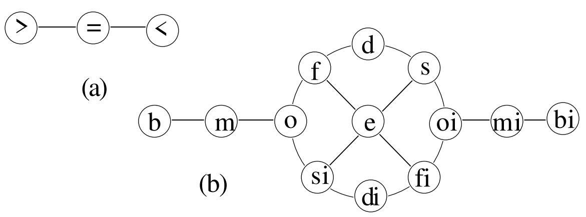

Figure 2. Neighborhood graphs for (a) instant-to-instant relations, (b) interval-to-interval relations. The neighborhood graph is made of relations as nodes and conceptual neighborhood as edges (reciprocal relationships are denoted with an "i" added at the end for the sake of readability).

The graph of figure 2a represents the graph of conceptual neighborhood N_3^A between instants (the only continuous deformation is translation). The graph of Figure 2b represents the conceptual neighborhood N_{13}^A for the deformation corresponding to the move of an extremity of an interval (more generally, the deformation corresponds to moving a limit). Throughout the paper, the only considered transformation A is the continuous move of a limit (called A-neighborhood in [9]). The influence of this choice is acknowledged when it matters.

2.4. Conversion from interval to instant formalisms

Relationships between intervals can be expressed in function of the relationships between their bounding instants (see table 4): any relationship between $x = \langle x^- x^+ \rangle$ and $y = \langle y^- y^+ \rangle$ is expressed by a quadruple (r_1, r_2, r_3, r_4) of relationships between the extremities defined as so:

$$\langle x^- x^+ \rangle (r_1, r_2, r_3, r_4) \langle y^- y^+ \rangle$$

$$\equiv x^- r_1 y^- \wedge x^- r_2 y^+ \wedge x^+ r_3 y^- \wedge x^+ r_4 y^+$$

considering that $x^- < x^+$ and $y^- < y^+$, each possible relationship between the bounding instants are expressible with such a quadruple (see table 4). The symbol ⇒ is used such that ⇒x is the expression of an interval as a couple of extremities and ⇒r a relationship between intervals expressed as a quadruple. ⇒ is extended towards sets of relations such that ⇒ρ is a set of quadruples. Thus:

$$< x^- x^+ > \left[\bigcup_{i=1}^{n} \{ (r_{i1}, r_{i2}, r_{i3}, r_{i4}) \} \right] < y^- y^+ >$$

$$\equiv \bigvee_{i=1}^{n} x^- r_{i1} y^- \wedge x^- r_{i2} y^+ \wedge x^+ r_{i3} y^- \wedge x^+ r_{i4} y^+$$

xry	$x^- r_1 y^-$	$x^- r_2 y^+$	$x^+ r_3 y^-$	$x^+ r_4 y^+$
b	<	<	<	<
d	>	<	>	<
o	<	<	>	<
s	=	<	>	<
f	>	<	>	=
m	<	<	=	<
e	=	<	>	=
m^{-1}	>	=	>	>
f^{-1}	<	<	>	=
s^{-1}	=	<	>	>
o^{-1}	>	<	>	>
d^{-1}	<	<	>	>
b^{-1}	>	>	>	>

Table 4. The 13 relationships between intervals expressed through relationships between interval extremities.

Since any formula representing relationship between four instants x^-, x^+, y^- and y^+ respecting the properties of intervals ($x^- < x^+$ and $y^- < y^+$) can be expressed under that form, the inverse operation ⇐ is defined. It converts such an expression between bounding instants of two intervals into a set of relations expressing the disjunction of relations holding between the intervals. Of course, both operators (⇐ and ⇒) are inverse.

3. Requirements for granularity change operators

We aim at defining operators for transforming the representation of a temporal situation from one granularity to another so that the resulting representation should be compatible with what can be observed under that granularity. The requirements for building such operators are considered here. The first section concerns what happens to classical models of time and to temporal entities when they are seen through granularity. The second one provides a set of properties that any system of granularity conversion operators should enjoy. These properties are expressed in a sufficiently abstract way for being meaningful for instants and periods, time and space.

3.1. Granularity change operators

Time is usually represented under a particular granularity. Thus, the time representation system presented so far is an adequate representation for time at any granularity (as far as only qualitative properties are considered). For instance, the three situations of figure 1 can be expressed in the same formalism with objects and qualitative relations between them. If we only consider the position of the objects along the horizontal line, the three elements (T, B and H) are related to each other in the way of figure 1c by T{m}B (the truck meets its bumper) and B{b}H (the bumper is before the house).

We aim at elucidating the relationship between two representations of the same reality under two different granularities. As a matter of fact, the situations of figure 1 cannot be merged into one consistent situation: figures 1b and c together are inconsistent since, in (b), B{m}H and, in (c), B{b}H which, when put together, gives B{}H. First, the reasons for these problems are examined before providing a set of properties that granularity change operators must satisfy.

Time is usually interpreted as a straight line, instants as points and intervals as segments. Under a numerical light, granularity can be defined as scaling plus filtering what is relevant and what is not (discretizing). However, granularity is a special filter since, as the name indicates, it filters on size. For the time concern, the granularity of a system can be defined as the duration of the smallest relevant event (relevance being defined independently beforehand). But what happens to non relevant events? There are two solutions:

- they can vanish;
- they can remain with size 0, i.e. as instants.

In both cases, these solutions share additional consequences (for symbolic representations): if, under a coarse granularity, one observes that some event is connected to another this can be wrong under a finer granularity since a non relevant laps of time could be relevant here. In another way, when communicating the same observation, it must be taken into account that the short laps of time may be non relevant (and thus that the relationship between the event can be disconnected). This is what happened for the relationship between B and H, which is {b}, in Figure 1c, and becomes {m}, in 1b.

In order to account for this situation, which appears to be regular, we need a downward (resp. upward) operator which, from a relationship observed a some particular granularity, is able to provide a set of relationships at a finer (resp. coarser) granularity which represents what *can* be perceived under that last granularity. The purpose here, is not to design granularity conversion operators which can make events vanish or turn into instants (see [5]) but rather operators which can account for the possibility of having, under a finer granularity, new space between two entities, and, vice-versa, that a space can become non relevant under a coarser granularity. These operators are called upward and downward granularity conversion operators and noted by the infix ${}_{g}{\uparrow}^{g'}$ and ${}^{g'}{\downarrow}_{g}$ operators (where g and g' are granularities such that g is finer — more precise — than g', i.e. that the size of relevant events is smallest in g than in g'). The following g→g' operator will be used for any of them when the property holds for both (then there is no constraint upon g and g'). As usual, the notation g→g' introduced for the conversion of a single relationship is extended towards sets:

$$g{\rightarrow}g'\ \rho = \bigcup_{r\in\rho} {}_{g}{\rightarrow}_{g'}\ r.$$

3.2. Properties for granularity change operators

Anyone can think about a particular set of such operators by imagining the effects of coarseness. But here are provided a set of properties which should be satisfied by any system of granularity conversion operators. In fact, the set of properties is very small. But next section shows that they are sufficient for constraining the possibility for such operators to only one (plus the expected operators corresponding to identity and conversion to everything).

Self-conservation

Self-conservation states that whatever be the conversion, a relationship must belong to its own conversion. It is quite a sensible and minimal property: the knowledge about the relationship can be less precise but it must have a chance to be correct.

(1) $r \in g{\rightarrow}g' r$ (self-conservation)

Neighborhood compatibility

A property considered earlier is the *order preservation property* [10] which states (a part of this):

$$x > y \Rightarrow \neg(g{\rightarrow}g'\ x < g{\rightarrow}g'\ y)$$ (order preservation)

However, this property has the shortcoming of being vectorial rather than purely topological. Its topological generalization, is *reciprocal avoidance*:

$$x \; r \; y \Rightarrow \neg(g{\rightarrow}g' \; x \; r^{-1} \; g{\rightarrow}g' \; y) \qquad \text{(reciprocal avoidance)}$$

Reciprocal avoidance, was over-generalized and caused problems with auto-reciprocal relationship (i.e. such that $r=r^{-1}$). The *neighborhood compatibility*, while not expressed in [5] has been taken into account informally: it constrains the conversion of a relation to form a conceptual neighborhood (and hence the conversion of a conceptual neighborhood to form a conceptual neighborhood).

(2) $\forall r, \forall r', r'' \in g{\rightarrow}g' r, \; \exists r_1, \ldots r_n \in g{\rightarrow}g' r$ such that $r_1 = r', \; r_n = r''$ and $\forall i \in [1, n-1] \;\; N_\Gamma^X(r_i, r_{i+1})$
(neighborhood compatibility)

This property has already been reported by Christian Freksa [9] who considers that a set of relationships must be a conceptual neighborhood for pretending being a coarse representation of the actual relationship. (2) is weaker than the two former proposals because it does not forbid the opposite to be part of the conversion, but, in such a case, it constrains whatever be in between the opposite to be in the conversion too. Neighborhood compatibility seems to be the right property, partly because, instead of the former ones, it does not forbid a very coarse grain under which any relationship is converted in the whole set of relations. It also seems natural because granularity can hardly be imagined as discontinuous (at least in continuous spaces).

Conversion-reciprocity distributivity

An obvious property for such an operator is symmetry. It is clear that the relationships between two temporal occurrences are symmetric and thus granularity conversion must respect this.

(3) $$(g{\rightarrow}g' \; \rho^{-1}) = (g{\rightarrow}g' \; \rho)^{-1}$$
(distributivity of $g{\rightarrow}g'$ on $^{-1}$)

Inverse compatibility

Inverse compatibility states that the conversion operators are consistent with each other, i.e. that, if the relationship between two occurrences can be seen as another relationship under some granularity, then the inverse operation from the latter to the former can be achieved through the inverse operator.

(4) $$r \in \bigcap_{r' \in {}_g\uparrow^{g'} r} {}^{g'}\!\downarrow_g r' \text{ and } r \in \bigcap_{r' \in {}^g\downarrow_{g'} r} {}_{g'}\!\uparrow^g r'$$
(inverse compatibility)

For instance, if someone in situation (b) of figure 1 is able to imagine that, under a finer granularity (say situation c), there is some space between the bumper and the house, then (s)he must be whiling to accept that if (s)he were in situation (c), (s)he could imagine that there is no space between them under a coarser granularity (as in situation b).

Cumulated transitivity

A property which is usually considered first is the full transitivity:

$$g{\rightarrow}g' \cdot g'{\rightarrow}g'' \; r = g{\rightarrow}g'' \; r$$

This property is too strong; it would for instance imply that:

$$g{\rightarrow}g' \cdot g'{\rightarrow}g \; r = r$$

Of course, it cannot be achieved because this would mean that there is no loss of information through granularity change: this is obviously false. If it were true anyway, there would be no need for granularity operators: everything would be the same under each granularity. We can expect to have the cumulated transitivity:

$${}_g\uparrow^{g'} \cdot {}_{g'}\uparrow^{g''} r = {}_g\uparrow^{g''} r \text{ and } {}^{g''}\!\downarrow_{g'} \cdot {}^{g'}\!\downarrow_g r = {}^{g''}\!\downarrow_g r$$

However, in a purely qualitative calculus, the amounts of granularity (g) are not relevant and this property becomes a property of idempotency of operators:

(5) $$\uparrow \cdot \uparrow = \uparrow \text{ and } \downarrow \cdot \downarrow = \downarrow \qquad \text{(idempotency)}$$

At first sight, it could be clever to have non idempotent operators which are less and less precise with granularity change. However, if this applies very well to quantitative data, it does not apply for qualitative: the qualitative conversion applies equally for a big granularity conversion and for a small one which is ten times less. If there were no idempotency, converting a relationship directly would give a different result than doing it through ten successive conversions.

Representation independence

Representation independence states that the conversion must not be dependent upon the representation of the temporal entity (as an interval or as a set of bounding instants). Again, this property must be required:

(6) $g{\rightarrow}g' \; \rho = \Leftarrow g{\rightarrow}g' \Rightarrow \rho$ and $g{\rightarrow}g' \; \rho = \Rightarrow g{\rightarrow}g' \Leftarrow \rho$
(representation independence)

Note that since $\Leftarrow$ requires that the relationships between bounding instants allows the result to be an interval, there could be some restrictions on the results (however, these restrictions correspond exactly to the vanishing of an interval that which is out of scope here).

4. The granular system for time relations

Once these six properties have been defined one can start generating candidate upward and downward conversion operators. However, these requirements are so precise that they leave no place for choice. We are showing below by starting with the instant algebra that there is only one possible couple of operators. Afterwards, this easily transfers to interval algebra.

4.1. Conversion operators for the instant algebra

The $64(=2^3.2^3)$ a priori possible operators for converting < and = can be easily reduced to six: the constraint (1) restricts the conversion of < to be {<}, {<=}, {<>} or {<=>} and that of {=} to be in {=}, {<=}, {=>} or {<=>}. The constraint (2) suppresses the possibility for < to become {<>}. The constraint (3) has been used in a peculiar but correct way for eliminating the {<=} (resp. {=>}) solutions for =. As a matter of fact, this would cause the conversion of $=^{-1}$ to be {=>} (resp. {<=}), but $=^{-1}$ is = and thus its conversion should be that of =.

< \ =	{=}	{<=>}
{<}	Id	α
{<=}	β	γ
{<=>}	δ	no info

Table 5. The six possible conversion operators for = and <.

There are still six possible conversion operators left (Id, α, β, γ, δ and NI). Since the above table does not consider whether the operators are for downward or upward conversion, this leaves, a priori, 36 upward-downward couples. But the use of property (4) — the putative operators must be compatible with their inverse operator (and vice-versa) — reduces them to 5: Id-Id, α-β, γ-γ, δ-δ and NI-NI.

The solution Id-Id cannot be considered as granularity since it does not provide any change in the representation. The solution NI-NI is such that it is useless. The δ-δ pair has the major flaw of not being idempotent (i.e. $\delta \cdot \delta \neq \delta$): as a matter of fact, the composition of δ with itself is NI, this is not a good qualitative granularity converter (this violates property 5). There are two candidates left: the γ-γ has no general flaw, it seems just odd to have an auto-inverse operator (i.e. which is its own inverse) since we all know the asymmetry between upward and downward conversion: it could be a candidate for upward conversion (it preserves the equality of equals and weakens the assertions of difference) but it does not fit intuition as a downward conversion operator (for the same reasons). Moreover, γ does not respect vectorial properties such as order-preservation (γ is just β plus the non distinction between < and >). Thus the α-β pair is chosen as downward/upward operators. The main argument in favor of α-β is that they fit intuition very well. For instance, if the example of figure 1 is modeled through bounding instants (x^- for the beginning and x^+ for the end) of intervals T^+, B^-, B^+ and H^-, it is represented in (c) by $T^+=B^-$ (the truck ends where the bumper begins), $B^-<B^+$ (the beginning of the bumper is before its end), $B^+<H^-$ (the end of the bumper is before the beginning of the house) in (b) by $B^+=H^-$ (the bumper ends where the house begins) and in (a) by $B^-=B^+$ (the bumper does not exist anymore). This is possible by converting with the couple α-β which allows to convert $B^+<H^-$ into $B^+=H^-$ ($= \in \beta<$) and $B^-=B^+$ into $B^-<B^+$ ($< \in \alpha=$), but not with the use of γ as a downward operator. Thus the following result is established:

PROPOSITION: The table 6 defines the only possible non auto-inverse upward/downward operators for A_3.

relation: r	$_g{\uparrow}^{g'} r$	$^{g}{\downarrow}_{g'} r$
<	< =	<
=	=	< = >
>	> =	>

Table 6. Upward and downward granularity conversions between instants.

The operators of table 6 also satisfy the properties of granularity operators.

PROPOSITION: The upward/downward operators for A_3 of table 6 satisfy the properties (1) through (5).

4.2. Conversion operators for the interval algebra

By constraint (6) the only possible operators for A_{13} are now given. They enjoy the same properties as the operators for A_3.

PROPOSITION: The upward/downward operators for A_{13} of table 7 are the only one to satisfy the property (6) with regard to the operators of A_3 of table 6.

PROPOSITION: The upward/downward operators for A_{13} of table 7 satisfy the properties (1) through (5).

The reader is invited to check on the example of figure 1, that what has been said about instant operators is still valid. The upward operator does not satisfy the condition (2) for B-neighborhood (violated by d, s and f) and C-neighborhood (o, s and f). This result holds since the corresponding neighborhoods are not based upon independent limit translations while this independence has been used for translating the results from A_3 to A_{13}.

relation: r	${}_{g}\!\uparrow^{g'} r$	${}_{g}\!\downarrow^{g'} r$
b	b m	b
d	d f s e	d
o	o f^{-1} s m e	o
s	s e	o s d
f	f e	d f o^{-1}
m	m	b m o
e	e	o f^{-1} d^{-1} s e s^{-1} d f o^{-1}
m^{-1}	m^{-1}	o^{-1} m^{-1} b^{-1}
f^{-1}	f^{-1} e	d^{-1} f^{-1} o
s^{-1}	s^{-1} e	d^{-1} s^{-1} o^{-1}
o^{-1}	o^{-1} s^{-1} f e m^{-1}	o^{-1}
d^{-1}	d^{-1} s^{-1} f^{-1} e	d^{-1}
b^{-1}	b^{-1} m^{-1}	b^{-1}

Table 7. Upward and downward conversion operators between intervals.

5. Granularity and inference

The composition of symbolic relationship is a favored inference mean for symbolic representation systems. One of the properties which would be interesting to obtain is the independence of the results of the inferences from the granularity level (property 7). The distributivity of $g \rightarrow g'$ on $\times$ denotes the independence of the inferences from the granularity under which they are worked out.

(7) $$g \rightarrow g' (\rho_1 \times \rho_2) = (g \rightarrow g' \rho_1) \times (g \rightarrow g' \rho_2)$$ (distributivity of $g \rightarrow g'$ over $\times$)

This property is only satisfied for upward conversion in A_3.

PROPOSITION: The upward operator for A_3 satisfies property (7).

It does not hold true for A_{13}: let consider three intervals x, y and z such that xby and ydz, the application of composition of relations gives x\{b o m d s\}z which, once upward converted, gives x\{b m e d f s o f^{-1}\}z. By opposition, if the conversion is first applied, it returns x\{b m\}y and y\{d f s e\}z which, once composed, gives x\{b o m d s\}z. The interpretation of this result is the following: by first converting, the information that there exists an interval y forbidding x to finish z is lost: if, however, the relationships linking y to x and z are kept, then the propagation will take this into account and recover the lost precision: \{b m e d f s o f^{-1}\}$\wedge$\{b o m d s\}=\{b o m d s\}. However, this cannot be enforced since, if the length of y is so small that the conversion makes it vanishing, the correct information at that granularity is the one provided by applying first the composition: x can meet z under such a granularity.

However, if (7) cannot be achieved for upward conversion in A_{13}, we proved that upward conversion is super-distributive over composition.

PROPOSITION: The upward operator for A_{13} satisfies the following property:

(8) $$({}_{g}\!\uparrow^{g'} \rho_1) \times ({}_{g}\!\uparrow^{g'} \rho_2) \subseteq {}_{g}\!\uparrow^{g'} (\rho_1 \times \rho_2)$$ (super-distributivity of ${}_{g}\!\uparrow^{g'}$ over $\times$)

A similar phenomenon appears with the downward conversion operators (it appears both for instants and intervals). So let consider three instants x, y and z such that $x>y$ and $y=z$, on one hand, the composition of relations gives $x>z$, which is converted to $x>z$ under the finer granularity. On the other hand, the conversion gives $x>y$ and $y\{<=>\}z$ because, under a more precise granularity y could be close but not really equal to z. The composition then provides no more information about the relationship between x and z ($x\{<=>\}z$). This is the reverse situation as before: it takes into account the fact that the indicernability of two instants cannot be ensured under a finer grain. Of course, if everything is converted first, then the result is as precise as possible: downward conversion is sub-distributive over composition.

PROPOSITION: The downward operators for A_{13} and A_3 satisfy the following property:

(9) $${}^{g}\!\downarrow_{g'} (\rho_1 \times \rho_2) \subseteq ({}^{g}\!\downarrow_{g'} \rho_1) \times ({}^{g}\!\downarrow_{g'} \rho_2)$$ (sub-distributivity of ${}^{g}\!\downarrow_{g'}$ over $\times$)

These two latter properties can be useful for propagating constraints in order to get out of them the maximum of information quickly. For instance, in the case of upward conversion, if no interval vanishes, every relationship must be first converted and then composed.

6. Further and ongoing works

Category theory which is widely used in programming language semantics has been introduced in knowledge representation [1] in order to account for the relation of approximation between, on the one hand, a knowledge base and the modeled domain, and on the other hand, the many achievements of a knowledge base. Ongoing works tackle the problem of such a categorical semantics for time representation. It meets the intuition: granular representation is approximation. This will provide the advantage of allowing the integration of a specialized time representation into a wider context (e.g. for adding temporal extension to objects represented as Ψ-terms). Category theory allows to do so in a general way.

7. Related works

Jerry Hobbs introduced the concept of granularity from the non discernability of particular terms with regard to a given set of predicates (these terms can be substituted in the range of any of the given predicates without changing their validity). The main difference here is that the granularity is given a priori in the structure of time and the scaling notion while Hobbs defines a granularity with regard to relevant predicates. To our knowledge there is no other proposal for integrating granularity into qualitative time representation.

There has been tremendous work on granularity in metric spaces. One of the more elaborate model is that of [11, 4]. It proposes a quantitative temporal granularity based on a hierarchy of granularities strictly constrained (to be convertible, divisible…) which offers upward and downward conversion operators for instants and intervals (instead of their relationships). [5] offers a more general (i.e. less constrained) framework for quantitative relationships and thus achieves weaker properties. Hence, the properties obtained here for qualitative representation are compatible with the quantitative representation of [11, 4]. Others works [3] considered granularity in a hybrid qualitative/quantitative system. The same effect as presented here could certainly been achieved through the computation of qualitative relationships from quantitative ones (using [11, 4] but not in a pure qualitative fashion.

[8] presents a systems which shares a great deal with ours: they treat granularity changes between several representations expressed in the same classical temporal logic (just like here, we used the classical A_3 and A_{13}) and they map these representations to natural numbers instead of real numbers [5]. However, temporal logics and algebra of relations are not immediately comparable so the results are quite different in nature. It is expected that the categorical framework sketched in §6 allows to compare the two approaches in depth.

8. Conclusion

In order to understand the relationships between several granularities, a set of requirements have been established for conversion operators. The only possible operators filling these requirements have been defined. Moreover other properties of the operators have been established (preservation of the relationship between points and interval). These operators can be used for combining information coming from different sources and overcoming their contradictory appearance. Further works have been done for extending qualitative granularity from time to space and are reported in [6, 7].

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